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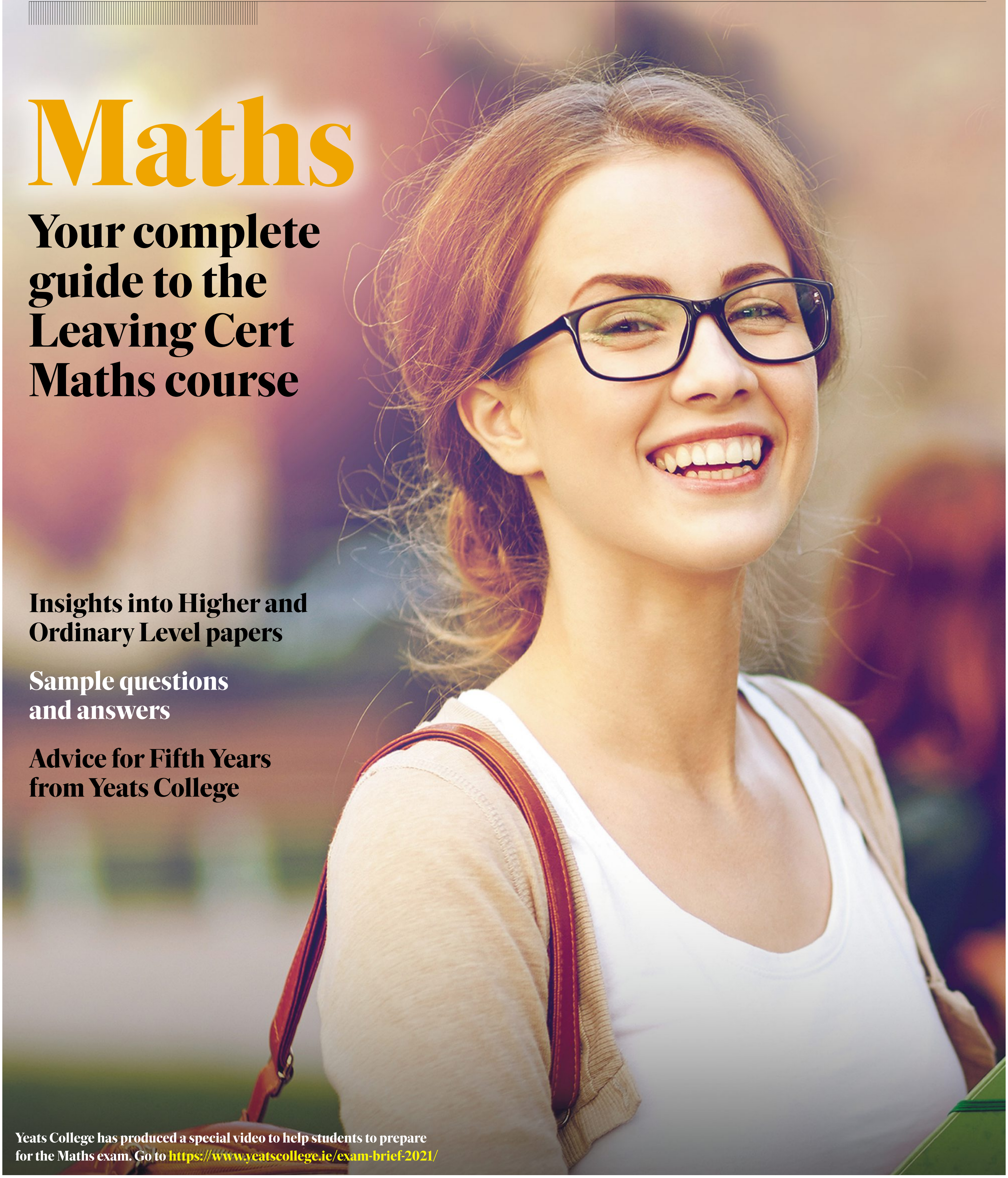
Exam Brief

Thursday, 29 April, 2021



Maths

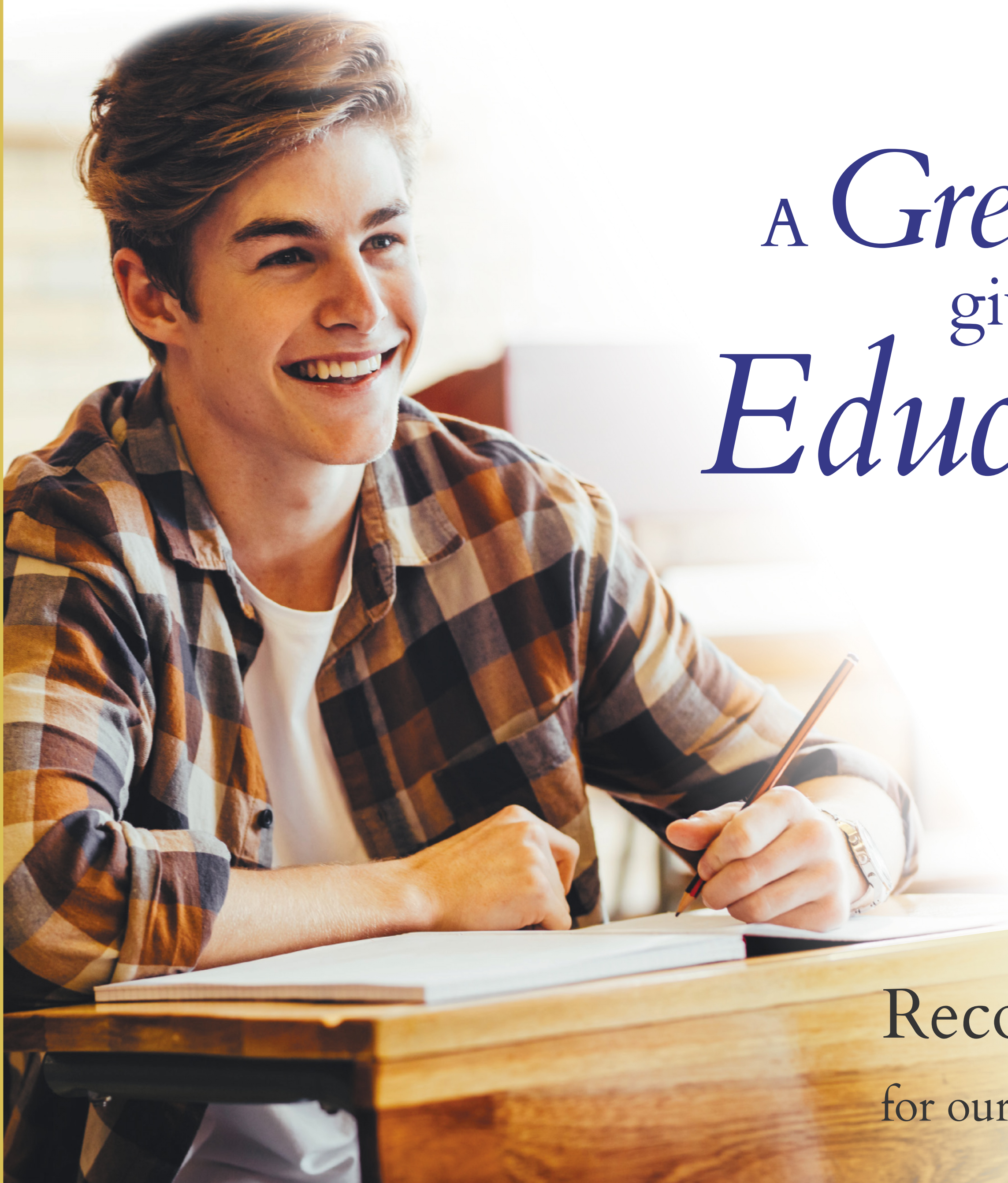
Your complete guide to the Leaving Cert Maths course

Insights into Higher and Ordinary Level papers

Sample questions and answers

Advice for Fifth Years from Yeats College

Yeats College has produced a special video to help students to prepare for the Maths exam. Go to https://www.yeatscollege.ie/exam-brief-2021/



A guiding hand on the Leaving Certificate journey

Welcome to the first in the *Exam Brief* series for the Leaving Cert in 2021.

Filled with expert advice and sample questions and answers, *Exam Brief* is the *Irish Independent*'s annual guide to the Leaving Cert. It is a must-read for every Leaving Cert student in the country and a helpful tool for parents keen to support their children during this time.

Once again, we are producing this series in collaboration with Yeats College, one of Ireland's leading independent schools, which consistently delivers outstanding exam results. The first in the series is today's 24-page guide for preparing for both the Higher Level and Ordinary Level Maths exams in the Leaving Cert.

Our contributors are all highly experienced teachers at Yeats College. They have provided some insightful advice around preparing for your exams as well as practical pointers on what to do while sitting each one. There is also advice for Fifth Years, bearing in mind what a challenging time they have also had because of the pandemic.

The supplements will continue in the coming two weeks, with English and Irish next week, followed by Other Subjects, which will include a number of subjects Leaving Cert students will welcome guidance on.

Make sure you collect your guide every Thursday in the *Irish Independent* and you will have the ideal accompaniment for your study and revision programme over the next few weeks.

SORCHA CORCORAN
Editor

Rising to the challenge

Founder and principal of Yeats College* Terry Fahy *reflects on this unprecedented time in the history of the Leaving Cert and applauds both teachers and students for their continued diligence

The arrival of the Covid-19 pandemic provided an immense challenge to all of us. From the depth of the challenges faced has emerged the incredible resilience and ingenuity of both the staff and students at Yeats College. During this time, we have learned that exceptional teaching and communication in a uniquely motivated environment will always embrace the student.

The real heroes are the Leaving Cert students of 2021, who have had an interrupted senior-cycle educational experience and, despite all the odds, will arrive in exam centres in June. The Class of 2021 is showing resolve and determination and the entire country supports them.

I have always believed that the best schools are consciously designed to nurture a desire for lifelong learning and discovery, to enable ambition as well as help to inspire their students with ideas. I believe Yeats College is such a place and I wish to pay tribute to the team of teachers at Yeats College, who each day create incredible learning experiences for our students.

As the lockdown became imminent and inevitable last year, Yeats College migrated its entire physical class timetable and related support systems to a completely online virtual school. From March 18, 2020, content-rich classes and guided learning took place in all subjects and continued right up to the conclusion of that final term.

Our classes were delivered through a variety of digital platforms embracing online teaching of all classes and supported with responsive monitoring of learning and homework. This year when we returned to school, in order to ensure continuity of learning, our classroom teaching was also live-streamed, allowing us to maintain progress and momentum for all throughout the year. Ultimately, this created a motivated and collaborative learning space where students maintained their learning structure to flourish academically throughout this challenging time.

I would like to welcome you to Part 1 of the *Irish Independent Exam Brief* series for 2021. As a Leaving Cert or Fifth Year student, you will benefit from the insights in these guides. They contain tips and advice from experienced Yeats College teachers. They are also designed to help parents to provide the best support for their children in the weeks ahead.

> 'The real heroes are the Leaving Cert students of 2021, who have had an interrupted senior-cycle educational experience and, despite all the odds, will arrive in exam centres in June'

Right now, as a student, you will be gearing up to complete a pivotal examination, which will shape your future. Completing the Leaving Cert is a key transitional moment, as it guides and has the potential to influence your future role in life. It certainly is an endurance test, but one that should be embraced. As with any apparently formidable challenge, once tackled constructively, it is much easier to overcome.

At Yeats College, our small size is an asset. It offers us flexibility in developing and delivering a rich, creative programme that addresses the academic, social and sporting needs of our student body. We are a community of focused, enthusiastic students and exceptional teachers and educators that embrace a passion to succeed.

The College is delighted to be involved with the *Irish Independent* in developing this educational resource. I would like to take this opportunity to thank the teaching staff in Galway and John Heffernan, principal of Yeats College Waterford, and his staff for their commitment to making Yeats College a unique and successful environment. I would also like to sincerely thank the *Irish Independent* team and editor Sorcha Corcoran in developing this resource.

Best of luck to you all and wishing you every success in your future.

TERRY FAHY
Founder and Principal
Yeats College

Yeats College

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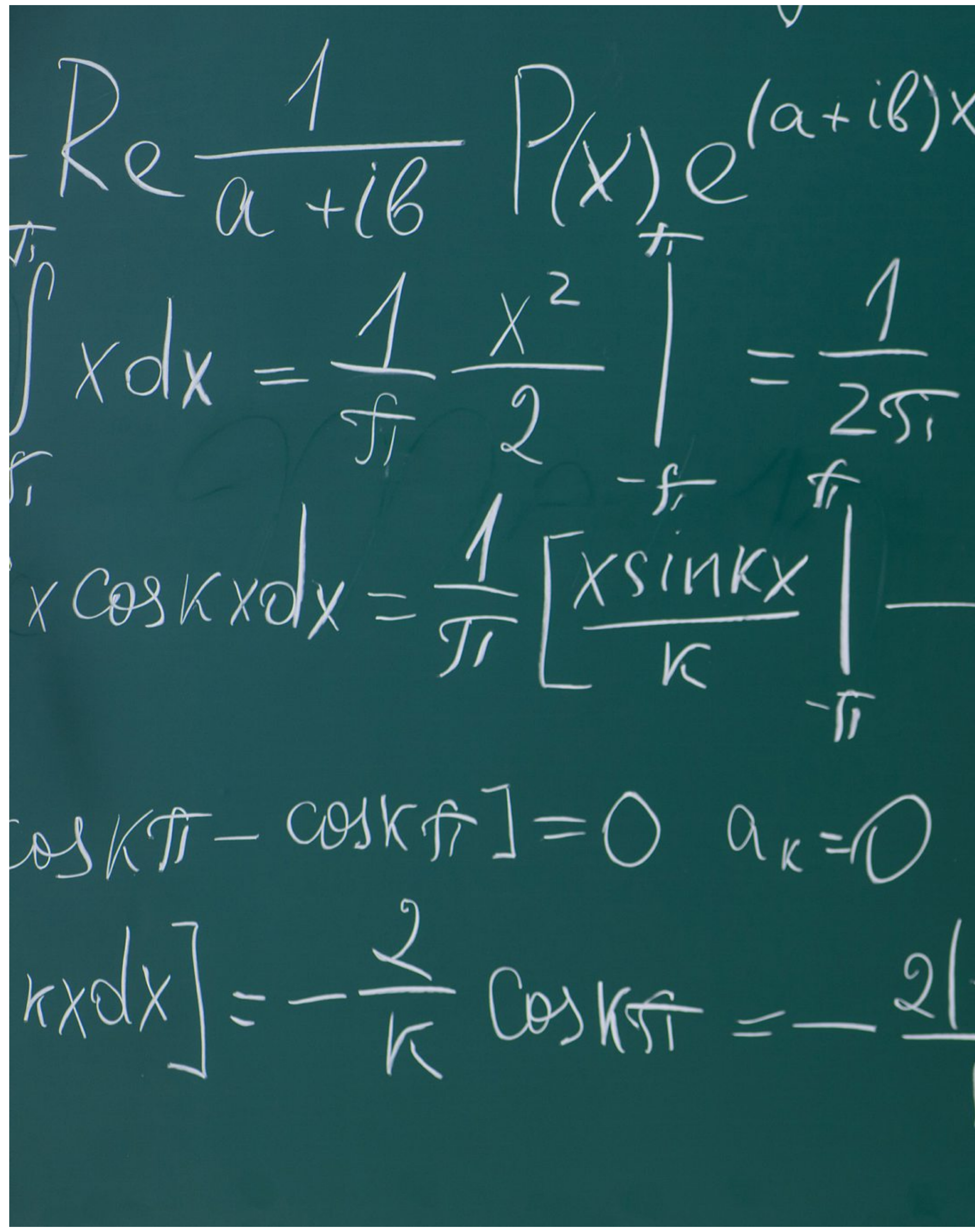
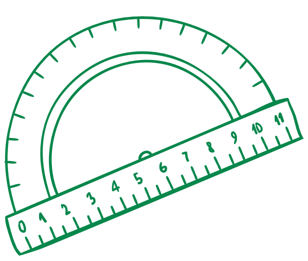
How a calculated approach can help you hit high figures in Maths



Maths teacher at Yeats College ***Dr Aisling Kenny-Naughton*** *highlights the key things to be aware of in the Higher Level Maths exam in this very different year*



Below I have listed my top tips on how to approach the Leaving Cert Maths exam. These tips are used by my students every year and have proven very effective. Different techniques work for different students. It's important to develop and practise your approach to the exams in advance of the Leaving Cert.

1 Keep calm
If you feel that a question is not going well, make sure to get as many attempt marks as possible. It is also important not to allow a bad question to have an effect on the rest of the paper. Move on and approach your next question with as much enthusiasm and confidence as your first.

2 Before answering a question, read it several times and pick out the important information
When you've completed the solution, re-read the question to ensure you have answered the question asked and given the answer in the correct format. For example, if you're asked for the coordinates of a point, make sure to find both x and y.

3 Be familiar with the marking scheme of past exam papers
Although attempt marks change from year to year and from paper to paper, it is helpful to have looked at marking schemes in advance. Students won't know how many marks a question is worth on the day. Do not assume a difficult question is worth a lot of marks.

4 Be very familiar with your calculator and the *Formulae and Tables Booklet*
In particular, you need to know how to reset your calculator, how to switch between degrees and radians and how to use table form. Be aware of what formulae are available in the booklet and where to find them. When studying, use your tables to find formulae so that you become more familiar with them.

5 There are a lot of good websites that can help you study
For example, *www.projectmaths.ie* contains a lot of resources, student activities, interactive files and quizzes. Also the website *www.scoilnet.ie* has resources organised by topic and is a very useful study aid.

6 Do not erase anything during the exam
If you make a mistake, simply put a line through your work. The examiner will look at all your attempts and give you the marks for the best attempt.

7 Show your workings
Present your work in a logical and neat way. Draw a diagram, sketch or graph where necessary.

8 Attempt every question
Do not leave any blanks in the exam. There are no marks awarded for a blank space so it's always worth your while to make an attempt.

9 Check your progress
If you struggle with a particular question while studying, look through the solution, then come back to it at a later date and try it again.

10 Expect the unexpected and prepare to be challenged
The Higher Level Maths exam will certainly contain difficult and challenging questions.

Present your work in a logical and neat way. Draw a diagram, sketch or graph where necessary

Staying motivated

Maths at higher level is a very long and difficult course. It can be hard to stay motivated while studying and revising. Work and revision must be consistent. Maths must be done every single day.

Here are a few techniques to remember:

- Be patient with your progress.
- Make sure you're making progress!
- Break the course into topics and create a study plan to cover all topics before the exam.
- Create a checklist of your topics and tick off items as you understand them.
- Practise, practise, practise!!

Exam technique

- Prepare and practise your exam techniques in advance. Find out what works for you. Do you prefer to answer Section A or Section B first? Most of my students prefer to find a question based on their favourite topic and answer that first. Others like to read through the entire paper first. Some just start at the first question and answer each question in order. In particular, for Leaving Cert 2021, it is important to think about your strategy in advance. With a choice on the paper and more time, students have more flexibility in how they approach the paper.
- Remember the questions are going to challenge everyone. If you're finding a question difficult, chances are every other student is finding it difficult too. Don't let a difficult question overwhelm you.
- Confidence is key!

Challenge accepted: The Higher Level Maths exam will contain difficult and challenging questions, so expect the unexpected

Marking scheme

Each marking scheme is based on a table of scales. This table changes from year to year and, at times, it even changes from Paper 1 to Paper 2. Although students know that a Section A question will be worth 30 marks, they don't know in advance how many marks are going for each part of the question. They also don't know how many attempt marks will be given. It is vital that students attempt everything. Attempt marks vary but at times they are quite generous.

Past paper analysis

The Project Maths syllabus was examined fully for the first time in 2015. Some topics still have not appeared on the exam.

Throughout the article, I will refer to topics that have never been examined before. It is important to pay attention to these topics and to prepare them for this year's exam.

Preparing for Section B

Students tend to struggle more with questions in Section B. The questions challenge even the strongest students, so be prepared for that. They also tend to combine several topics into one question. Students need to practise questions that involve many topics. For example, solving a question on the circle could involve a mixture of coordinate geometry, trigonometry and geometry.

Read the questions very carefully and several times. Then, look at the keywords in the question and try to pick out the important points

Read the questions very carefully and several times. It is very important always to first figure out what the question being asked is. Then, look at the keywords in the question and try to pick out the important points. Write the points using Maths terms. For example, if the question asks for a maximum, you should know that you will need to differentiate. Or if you are given the rate of change of a volume, you should know to write this as $\frac{dV}{dt}$.

It is very important to prepare for this section by practising questions. Practise using past papers and your textbook. These questions require problem-solving skills that take time to develop. Start now and be patient with your progress.

Advice for Fifth Years

Fifth-Year students need to be very aware that this is a very long and difficult course. Students need to work hard across the two-year programme in order to be successful.

- Revise any topics covered in school by completing the relevant past exam paper questions.
- Work hard now to perfect your basic algebraic skills. Make sure you know how to factorise, deal with fractions and solve equations. It is important that you don't leave it until Sixth Year to develop these skills. You will be using algebra to solve problems on a regular basis and you need to be able to do this easily.
- Trigonometry is a massive topic for Paper 2. If you have covered Trigonometry in school, spend time revising it. The skills used will be built upon next year so they need to be perfect.
- Test yourself! Review any class tests from throughout Fifth Year. Retry questions that you did not answer correctly in the exam and mark yourself. Make sure you are improving!
- Consistency is as important in Fifth Year as in Sixth Year. Do not wait until Sixth Year to start revision. Do a little bit every day throughout the two-year cycle.

Exam structure

There are two Maths papers. Each paper lasts 2 hours, 30 minutes and, for this year, each paper will be worth 220 marks.

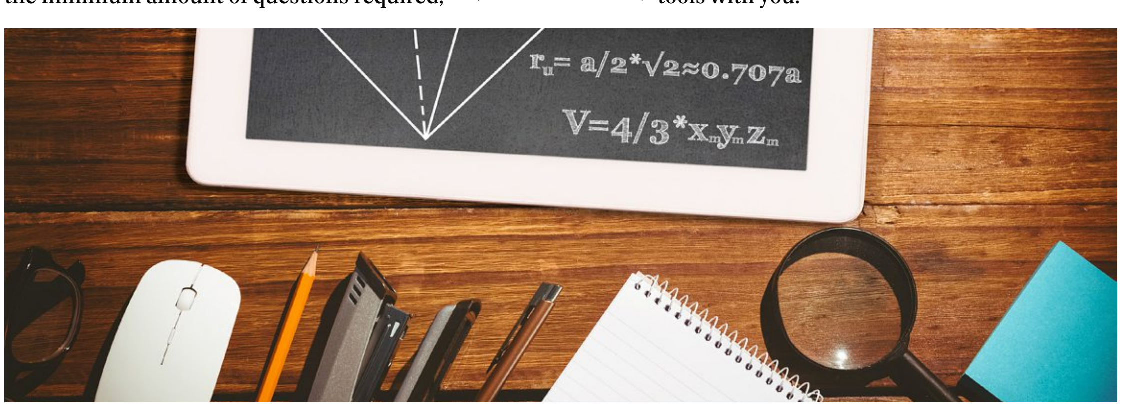
Each paper has two sections.

Section A: Concepts and Skills will consist of six questions worth 30 marks each. The student will be required to answer four of these. This section examines your basic maths skills.

Section B: Contexts and Applications will consist of four questions worth 50 marks in total. The student will be required to answer two of these. In this section, students have the extra challenge of dealing with a word problem.

The timing for the exam is usually very straightforward. In previous years, the student would simply divide the marks by 2 to find the time allowed for it. A 50-mark question should take 25 minutes. However, this year is different. If you intend to answer the minimum amount of questions required, you should spend 32 minutes on a Section A question and 55 minutes on a Section B question. This would leave you with 12 minutes to look over your paper. However, my advice is to stick to the time limits from previous years. Spend 15 minutes on each Section A question and 25 minutes on each Section B question. This now leaves you with 40 minutes extra to answer additional questions. This ensures you get the most out of the exam.

Be aware that some topics can appear on either paper. For example, Area & Volume is a topic required for Paper 2. However, it can appear on Paper 1 in the form of a max/min question or an area question. You need to be prepared for it to appear on either paper. Similarly, trigonometric functions can appear on either paper. Finding the slope of a tangent to a circle has never been examined and this topic could appear on either paper. Also, you may be asked to do a construction in Paper 1 (construct $\sqrt{2}$ or $\sqrt{3}$) so have the necessary tools with you.



Paper One

There are very few things that we need to learn off by heart in Maths as we mostly use skills to solve problems. However, there are some things that you will need to know that require **learning**. Make sure you learn the following for Paper 1.

- How to construct $\sqrt{2}$ and $\sqrt{3}$
- The proof that $\sqrt{2}$ is irrational
- If $x - a$ is a factor of *f(x)* then *a* is a root of the function. i.e. *f(a)=0*
- The following formulae for factorising are not in the *Formulae and Tables Booklet*

$$x^2 - y^2 = (x-y)(x+y)$$

$$x^3 - y^3 = (x-y)(x^2+xy+y^2)$$

$$x^3 + y^3 = (x+y)(x^2-xy+y^2)$$

- To form a quadratic equation from its roots:

$$x^2 - (\text{sum of the roots})x + (\text{product of the roots}) = 0$$

- Interpret roots of quadratic functions: For the function

$$f(x) = ax^2 + bx + c:$$

  - If $b^2 - 4ac > 0$: the function has two distinct, real roots.
  - If $b^2 - 4ac = 0$: the function has two equal, real roots.
  - If $b^2 - 4ac \geq 0$: the function has two real roots.
  - If $b^2 - 4ac < 0$: the function has two complex roots.

- To solve modulus inequalities:

If $|x| < a: -a < x < a$

If $|x| > a: x < -a$ and $x > a$

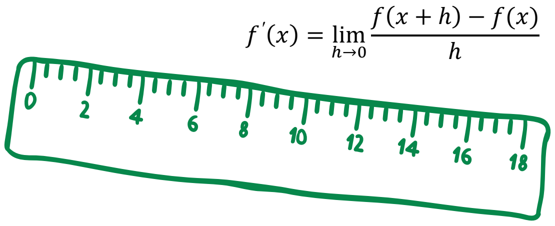
- Induction proofs including the proof of De Moivre's Theorem by induction for $\in N$.
- Derive the formula for the sum to infinity of geometric series by considering the limit of a sequence of partial sums.
- Learn the formula for the sum to infinity of a geometric series. It appears regularly on Paper 1. If

$$|r| < 1$$

$$S_\infty = \frac{a}{1-r}$$

- Derive the formula for a mortgage repayment.
- Definitions of injective, surjective and bijective functions.
- The definition required for differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$



- Sketching curves: A curve is increasing when $\frac{dy}{dx} > 0$

A curve is decreasing when $\frac{dy}{dx} < 0$

At a stationary/turning point $\frac{dy}{dx} = 0$

A curve is concave upwards when $\frac{d^2y}{dx^2} > 0$

A curve is concave downwards when $\frac{d^2y}{dx^2} < 0$

At a local maximum $\frac{dy}{dx} = 0$ *and* $\frac{d^2y}{dx^2} < 0$

At a local minimum $\frac{dy}{dx} = 0$ *and* $\frac{d^2y}{dx^2} > 0$

At a point of inflection $\frac{d^2y}{dx^2} = 0$

- This formula is examined regularly and must be **learnt off**. The average value of a function *y = f(x)* over an interval [*a*, *b*] is:

$$\frac{1}{b-a}\int_a^b y\,dx$$


Yeats College students

Algebra

An indepth knowledge of algebra is vital. It appears on Paper 1 but it is used in many questions to help solve problems. The concepts involved need to be well understood by higher-level students. We expect to see a question on algebra skills in Section A. However, it can also appear in Section B as a word problem.

- Be familiar with the number systems N, Z, Q, R, C and their elements.
- Construct $\sqrt{2}$ and $\sqrt{3}$
- Prove that $\sqrt{2}$ is not rational
- **Scientific notation:** Be able to work with numbers in the form $a \times 10^n, n \in \mathbb{Z}, 1 \leq a < 10.$

You may be asked to leave an answer in this form.

- **Logs & Indices:** Be very familiar with the rules on page 21 of the formulae booklet. Practise changing between an index equation and a log equation. *Remember:* if your variable is a power, you must use logs to solve the equation!

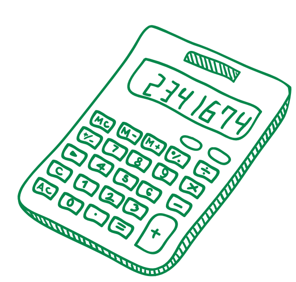
Example: Solve $2^{5x} = 50$

Turn into a log equation:

$$5x = \log_2 50$$

$$x = \frac{1}{5}\log_2 50$$

$$x = 1.13$$



- Simplify and evaluate algebraic expressions
- Simplifying algebraic fractions

Example: Write as a single fraction in its simplest form.

$$\frac{1}{x+2} - \frac{3}{x-3} + \frac{4}{x^2-x-6}$$

First, in order to find the common denominator, we need to factorise:

$$= \frac{1}{x+2} - \frac{3}{x-3} + \frac{4}{(x+2)(x-3)}$$

$$= \frac{(x-3) - 3(x+2) + 4}{(x+2)(x-3)}$$

$$= \frac{x - 3 - 3x - 6 + 4}{(x+2)(x-3)}$$

$$= \frac{-2x-5}{(x+2)(x-3)}$$

- **Binomial Theorem:** this was added to the most updated version of the syllabus and **has never been examined.** Learn that the $(r+1)$ st term of a binomial expansion of $(x+y)^n$ is given by:

$$\binom{n}{r} x^{n-r} y^r$$

This formula is on page 20 of the *Formulae and Tables Booklet*.

Example: Find the value of the term that is independent of x in the expansion

$$\left(x+\frac{1}{x}\right)^8$$

The general term is given by

$$\binom{8}{r}(x)^{8-r}\left(\frac{1}{x}\right)^r$$

This can be simplified to:

$$\binom{8}{r}(x)^{8-r}\left(\frac{1}{x^r}\right)$$

$$=\binom{8}{r}\left(\frac{x^{8-r}}{x^r}\right)$$

For a term to be independent of x, the powers of the x above and below the line must be equal.

Therefore,

$$8-r=r$$
$$2r=8$$
$$r=4$$

Subbing this value into the general term, we get

$$\binom{8}{4}(x)^4\left(\frac{1}{x}\right)^4$$
$$=\binom{8}{4}$$
$$=70$$

Therefore, the value of the term that is independent of x is 70.

▶ **Factorise:** Be able to use different methods of factorising. Remember to always look for a common factor first.

Example: Factorise fully x^4-81

Using difference of two squares:

$$x^4-81=(x^2-9)(x^2+9)$$

To fully factorise, we need to use difference of two squares again to get:

$$(x-3)(x+3)(x^2+9)$$

▶ Rearrange formulae

Example: Write c as the subject of the formula:

$$\frac{1}{c}+\frac{1}{b}=\frac{1}{a}$$

First, multiply across by the common denominator abc:

$$ab+ac=bc$$

Collect all the terms that contain c

$$bc-ac=ab$$
$$c(b-a)=ab$$
$$c=\frac{ab}{b-a}$$

▶ Solve the following types of equations using various strategies including algebraically and graphically:

● **Linear:**

When solving linear equations, collect variables on one side of the equation and constants on the other.

Example: Solve

$$3(x-5)=5(x+3)+3x$$
$$3x-15=5x+15+3x$$
$$3x-5x-3x=15+15$$
$$-5x=30$$
$$x=-6$$

● **Rational:**

Example: Solve

$$\frac{5x-3}{2}+\frac{3x-2}{7}=7$$

Multiply across by the common denominator 14:

$$7(5x-3)+2(3x-2)=7(14)$$
$$35x-21+6x-4=98$$
$$41x-25=98$$
$$41x=123$$
$$x=3$$

● **Quadratic:**

There are a few approaches to solving a quadratic equation. Firstly, bring all variables to one side of the equation. If you can factorise the expression, set each factor equal to zero and solve. Otherwise, use the formula for solving a quadratic. Do not waste time looking for factors! If struggling, use the formula.

Example: Solve

$$4x^2-8x+1=0$$

leaving your answer in surd form.

$$x=\frac{-(-8)\pm\sqrt{(-8)^2-4(4)(1)}}{(2)(4)}$$
$$x=\frac{8\pm\sqrt{48}}{8}$$
$$x=\frac{8\pm4\sqrt{3}}{8}$$
$$x=1\pm\frac{\sqrt{3}}{2}$$

● **Simultaneous equations with two variables:**

Example: Solve for x and y:

$$3x+2y=3$$
$$2x-3y=-11$$

$$A: 3x+2y=3$$
$$\underline{B: 2x-3y=-11}$$

$$3A: 9x+6y=9$$
$$\underline{2B: 4x-6y=-22}$$

$$13x=-13$$
$$x=-1$$

Sub this value into either of the original equations to find y:

$$3(-1)+2y=3\Rightarrow y=3$$

Remember to always find the second variable.

● **Simultaneous equations with three variables:**

Example: Solve for x, y and z:

$$A: x-y+z=4$$
$$B: 2x-3y+z=11$$
$$C: x-2y-2z=9$$

Eliminate z from two pairs of equations:

$$A: x-y+z=4$$
$$\underline{-B:\ -2x+3y-z=-11}$$
$$-x+2y=-7$$

$$2B: 4x-6y+2z=22$$
$$\underline{C: x-2y-2z=9}$$
$$5x-8y=31$$

Now, use this pair of equations to eliminate another variable

$$-x+2y=-7$$
$$\Rightarrow -5x+10y=-35$$
$$-5x+10y=-35$$
$$\underline{5x-8y=31}$$
$$2y=-4$$
$$y=-2$$
$$x=3$$
$$z=-1$$

● **Simultaneous equations with one linear and one non-linear equation:**

Example:

Solve for x and y:

$$x^2+y^2-6x+10y+25=0$$
$$x+y=1$$

(In this question, we are finding the points of intersection of a line and a circle)

Re-write the linear equation:

$$y=1-x$$

Substitute into the non-linear equation:

$$x^2+(1-x)^2-6x+10(1-x)+25=0$$
$$x^2+1-2x+x^2-6x+10-10x+25=0$$
$$2x^2-18x+36=0$$
$$x^2-9x+18=0$$
$$(x-6)(x-3)=0$$
$$x=6 \text{ or } \quad x=3$$

Since $\quad y=1-x$

therefore

$y=-5$ or $\quad y=-2$

$(6,\quad -5)(3,-2)$

● **Use the factor theorem for polynomials including solving cubic equations:**

Example: Solve

$$x^3-5x^2+5x-1=0$$

Use trial and error to find a root:

$(1)^3-5(1)^2+5(1)-1=0\ \Rightarrow x=$ 1 is a root.

$\Rightarrow x-1$ is a factor so we can divide by $x-1$

From long division, we find that:

$$x^3-5x^2+5x-1$$
$$=(x-1)(x^2-4x+1)=0$$

Since we cannot find factors for

$$x^2-4x+1,$$

we must solve it using the formula:

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(1)}}{(2)(1)}$$
$$x=\frac{4\pm\sqrt{12}}{2}$$
$$x=2\pm\sqrt{3}$$
$$\Rightarrow x=1, 2+\sqrt{3}, 2-\sqrt{3}$$

▶ **Form equations from the roots**

To form a quadratic equation from its roots:

$$x^2-(sum of the roots)x$$
$$+(product of the roots)=0$$

Example: Form the quadratic equation with roots $3+\sqrt{5}$ and $3-\sqrt{5}$

Sum of the roots =

$$3+\sqrt{5}+3-\sqrt{5}\ =6$$

Product of the roots =

$$(3+\sqrt{5})(3-\sqrt{5})=4$$

Therefore the quadratic equation is:

$$x^2-6x+4=0$$

▶ **Interpret roots of quadratic equations**

If $b^2-4ac>0$: the equation has two distinct, real roots.

If $b^2-4ac=0$: the equation has two equal, real roots.

If $b^2-4ac<0$: the equation has two complex roots.

Example: Prove that the equation

$$x^2+kx+(2k-4)=0$$

has real roots for all values of $k\in\mathbb{R}$

$$b^2-4ac=k^2-4(1)(2k-4)$$
$$=k^2-8k+16$$
$$=(k-4)^2$$
$$\geq 0$$

for all k.

▶ **Solve inequalities: linear, quadratic, rational, modulus and general.**

Example: Solve the inequality for

$x\in\mathbb{R},\ \ x\neq-2$

$$\frac{3x-1}{x+2}\geq 2$$

Learn the approach to this question. Students often make mistakes here.

Multiply both sides by $(x+2)^2$

$$\frac{(3x-1)(x+2)^2}{x+2}\geq 2(x+2)^2$$
$$(3x-1)(x+2)\geq 2(x^2+4x+4)$$
$$3x^2+5x-2\geq 2x^2+8x+8$$
$$x^2-3x-10\geq 0$$

We now have a quadratic inequality to solve. First, find where the function crosses the X–axis:

$$x^2-3x-10=0$$
$$(x-5)(x+2)=0$$
$$x=5,-2$$

Then, from the shape of the function we can see that

$$x^2-3x-10\geq 0$$

when

$$x\leq -2 \text{ and } x\geq 5$$

However, $x\neq -2$ so the solution is

$$x<-2 \text{ and } x\geq 5$$

● **To solve modulus inequalities, use the fact that**

- $|x|<1$ means $-1<x<1$
- $|x|>1$ means $x<-1$ and $x>1$

Example:

Solve the inequality $|3x+2|<5$

This means that:

$$-5<3x+2<5$$
$$-7<3x<3$$
$$-\frac{7}{3}<x<1$$

Complex Numbers

Complex Numbers have only ever appeared in Section A. However, this does not mean that a Section B question is impossible. Questions generally involve polar form and the use of De Moivre's theorem.

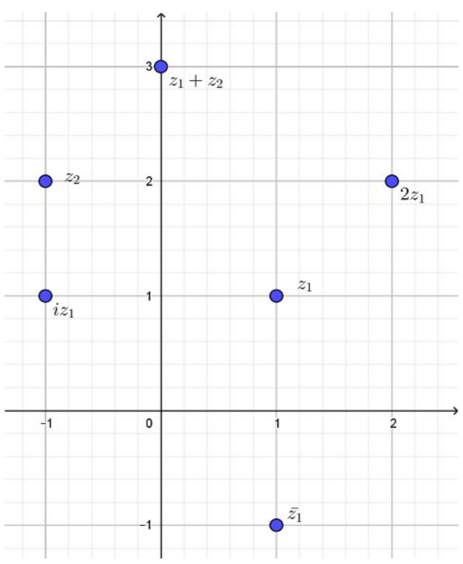
- Be able to work with complex numbers in rectangular form. It's important to be able to plot numbers on the Argand diagram.

Example: If $z_1 = 1 + i$ and $z_2 = -1 + 2i$, plot the following:

$$z_1 + z_2, 2z_1, \overline{z_1}, iz_1$$

Note in the diagram that:

- $0 + 0i$, z_1, z_2 and $z_1 + z_2$ form a parallelogram.
- The points $0 + 0i$, z_1 and $2z_1$ are collinear.
- When z_1 is rotated 90° anti-clockwise about the origin, you get iz_1



- Be able to calculate the modulus $|z|$ of a complex number z. Remember that the modulus of z means the distance of z from the origin. In general, if $z = x + yi$, then the modulus of z is found using the rule:

$$|z| = \sqrt{x^2 + y^2}$$

- Be able to calculate the conjugate $\bar{z}$ of a complex number z.

Example: Given the complex number $z = -1 + i$, then its conjugate $\bar{z} = -1 - i$.

- Be able to solve equations with real or imaginary coefficients. Be familiar with examples that involve the following:

→ A **quadratic equation** with **real** coefficients either has two real roots **or** two complex roots that are conjugates of each other.

→ A **cubic equation** with **real** coefficients either has three real roots **or** one real root and two complex roots that are conjugates of each other.

Example: Find the cubic equation whose roots are $2 \pm 3i$ and -1.

First, find the quadratic formed by the roots $2 \pm 3i$

Sum of the roots =
$$2 + 3i + 2 - 3i = 4$$

Product of the roots $= (2 + 3i)(2 - 3i)$
$$= 4 - 9i^2$$
$$= 13$$

We find the quadratic using
$$z^2 - z\,(\textit{sum of the roots}) + \textit{product} = 0$$
$$z^2 - 4z + 13 = 0$$

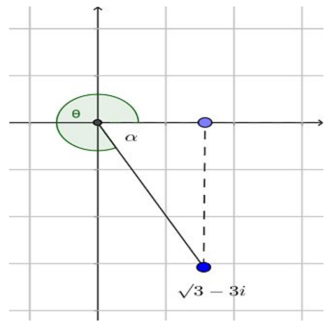
Now, form the cubic:
$$(z + 1)(z^2 - 4z + 13) = 0$$
$$z^3 - 4z^2 + 13z + z^2 - 4z + 13 = 0$$
$$z^3 - 3z^2 + 9z + 13 = 0$$

- Be able to put complex numbers into polar form and use De Moivre's theorem.

Example: Write $z = \sqrt{3} - 3i$ in polar form

$$|z| = \sqrt{\left(\sqrt{3}\right)^2 + (-3)^2}$$
$$= \sqrt{12} = 2\sqrt{3}$$

To find the argument: first we need to find the angle α so that we can then find θ as shown in the diagram.



$$\tan\alpha = \frac{3}{\sqrt{3}} \Rightarrow \alpha = \frac{\pi}{3}$$
$$\Rightarrow \theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$$
$$z = 2\sqrt{3}\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$$

Understand the difference in the questions below.

Example 1: Evaluate z^2 where
$$z = 2 + 2\sqrt{3}i$$

First, write z in polar form
$$z = 4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$

Using De Moivre's theorem
$$z^2 = (4)^2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$
$$z^2 = 16\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$$

Example 2: Solve $z^2 = 2 + 2\sqrt{3}i$
$$z^2 = 2 + 2\sqrt{3}i \Rightarrow z = \left(2 + 2\sqrt{3}i\right)^{\frac{1}{2}}$$

N.B. For solving an equation, we need general polar form
$$z^2 = 4\left[\cos\left(\frac{\pi}{3} + 2n\pi\right) + i\sin\left(\frac{\pi}{3} + 2n\pi\right)\right]$$
$$z = \sqrt{4}\left[\cos\left(\frac{\pi}{6} + n\pi\right) + i\sin\left(\frac{\pi}{6} + n\pi\right)\right]$$
$$z = 2\left[\cos\left(\frac{\pi}{6} + n\pi\right) + i\sin\left(\frac{\pi}{6} + n\pi\right)\right]$$

As there are going to be two different solutions, sub in two consecutive values for n to find the solutions.
$$z_1 = 2\left[\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right]$$
$$= \sqrt{3} + i$$
$$z_2 = 2\left[\cos\left(\frac{\pi}{6} + \pi\right) + i\sin\left(\frac{\pi}{6} + \pi\right)\right]$$
$$= -\sqrt{3} - i$$

- Be able to prove De Moivre's Theorem by induction for $n \in N$
- Use applications such as identities such as

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

These identities **have never been examined.**

Example: Use de Moivre's Theorem to prove that
$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$

We start by expanding
$$(\cos\theta + i\sin\theta)^3$$

using the binomial theorem:
$$(\cos\theta + i\sin\theta)^3 =$$
$$\cos^3\theta + 3\cos^2\theta\,(i\sin\theta)$$
$$+ 3\cos\theta\,(i\sin\theta)^2 + (i\sin\theta)^3$$
$$= \cos^3\theta + 3i\cos^2\theta\sin\theta$$
$$+ 3i^2\cos\theta\sin^2\theta + i^3\sin^3\theta$$
$$= \cos^3\theta + 3i\cos^2\theta\sin\theta$$
$$-3\cos\theta\sin^2\theta - i\sin^3\theta$$
$$= (\cos^3\theta - 3\cos\theta\sin^2\theta)$$
$$+ i(3\cos^2\theta\sin\theta - \sin^3\theta)$$

We also use De Moivre's Theorem to write the expression as follows:
$$(\cos\theta + i\sin\theta)^3$$
$$= \cos 3\theta + i\sin 3\theta$$

Equate the imaginary parts:
$$\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta$$

Replace $\cos^2\theta$ with $1 - \sin^2\theta$:
$$\sin 3\theta = 3(1 - \sin^2\theta)\sin\theta - \sin^3\theta$$
$$\sin 3\theta = 3\sin\theta - 3\sin^3\theta - \sin^3\theta$$
$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$

Sequences and Series

Sequences and Series can appear in many different forms on Paper 1. They have appeared in both Section A and Section B in the past. Also, the applications of them have been asked, including financial applications.

- Students need to be able to recognise a sequence and identify the type of sequence being used. Remember:
  → if the first change is constant, the sequence is arithmetic
  → if the second change is constant, the sequence is quadratic
  → if ratios are constant, the sequence is geometric.

Example: Prove that the sequence
$$T_n = \tfrac{2}{5}n + \tfrac{3}{2} \text{ is arithmetic.}$$

A sequence is arithmetic if
$$T_{n+1} - T_n = d \text{ where } d \text{ is the constant change.}$$

Since $T_n = \frac{2}{5}n + \frac{3}{2}$, we can find
$$T_{n+1} = \tfrac{2}{5}(n + 1) + \tfrac{3}{2}$$
$$T_{n+1} - T_n =$$
$$\left[\frac{2}{5}(n + 1) + \frac{3}{2}\right] - \left[\frac{2}{5}n + \frac{3}{2}\right]$$
$$= \frac{2}{5}n + \frac{2}{5} + \frac{3}{2} - \frac{2}{5}n - \frac{3}{2}$$
$$= \frac{2}{5}$$
$$T_{n+1} - T_n = \tfrac{2}{5}, \text{ a constant,}$$

therefore the sequence is arithmetic.

- Be familiar with arithmetic sequences and series and the T_n and S_n formulae. They are on page 22 of the *Formulae & Tables booklet*.

Example: Jack starts a new training schedule. He runs 10km in his first week of training and after that he runs an extra 2km each week

(a) How many kilometres does he run during his fifth week?

Firstly, write out the sequence:
10, 12, 14, …

Since the change from term to term is constant, we know the sequence is arithmetic.

To find a term of an arithmetic sequence, use the formula:
$$T_n = a + (n - 1)\,d$$

We are looking for T_5, so we substitute the values $a = 10$, $d = 2$, $n = 5$:
$$T_5 = 10 + (4)(2) = 18\text{km}$$

(b) How much does he run in total during the first five weeks of training?

Here we are looking for S_5 so we use the formula:
$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

Again, we substitute the values
$$a = 10, d = 2, n = 5$$
$$S_5 = \frac{5}{2}[(2)(10) + (4)(2)] = 70\ km$$

- You need to know how to calculate the T_n for a quadratic sequence.

Example: Find T_n for the sequence .
3, 6, 11, 18, …

The second change is constant therefore the sequence is quadratic. So T_n is of the form $an^2 + bn + c$ where a equals half the second change.

Therefore,
$$T_n = n^2 + bn + c$$
$$T_1 = (1)^2 + b\,(1) + c = 3$$
$$\rightarrow b + c = 2$$
$$T_2 = (2)^2 + b(2) + c = 6$$
$$\rightarrow 2b + c = 2$$

Solve these simultaneous equations to find that $b = 0$, $c = 2$
$$\rightarrow T_n = n^2 + 2$$

- Be familiar with geometric sequences and series and the formulae for T_n and S_n for these sequences. They are on page 22 of the *Formulae & Tables booklet*.

Example: The second term of a geometric sequence is 21. The third term is –63.

Find the first term and the common ratio.

Our sequence is: a, 21, –63, …

Therefore, the common ratio is
$$\frac{-63}{21} = -3$$

Therefore, $a = \frac{21}{-3} = -7.$

- Students also need to solve problems involving infinite geometric series. **Learn** the formula for S_∞ given below as it appears regularly on the paper.

Example: Find the sum to infinity for the sequence
$$3, \tfrac{3}{2}, \tfrac{3}{4}, \ldots$$

For this sequence,
$$a = 3, r = \frac{1}{2};$$
$$S_\infty = \frac{a}{1 - r} = \frac{3}{1 - \frac{1}{2}} = \frac{3}{\frac{1}{2}} = 6$$

- Students need to be able to apply their knowledge to problems involving recurring decimals

Example: Write $0.\dot{6}\dot{3}$ as a fraction, using the sum of a geometric series.
$$0.\dot{6}\dot{3} = \frac{63}{100} + \frac{63}{10000} + \cdots$$

This series is an infinite geometric series with
$$a = \frac{63}{100}, r = \frac{1}{100};$$
$$S_\infty = \frac{a}{1 - r} = \frac{\frac{63}{100}}{1 - \frac{1}{100}} = \frac{\frac{63}{100}}{\frac{99}{100}} =$$
$$= \frac{63}{99} = \frac{7}{11}$$

- Be able to derive the formula for the sum to infinity of geometric series by considering the limit of a sequence of partial sums.

Yeats College students

Functions

Functions and calculus are very important topics for Paper 1. They regularly appear in both sections and have both been examined several times on the same paper.

Students need to be very familiar with these topics.

▶ Limits: Be able to evaluate limits

Example 1: As $n \to k$

Evaluate $\lim_{x\to 3} \frac{x^2-9}{x-3}$

$$\lim_{x\to 3} \frac{x^2-9}{x-3} = \lim_{x\to 3} \frac{(x-3)(x+3)}{x-3}$$

$$= \lim_{x\to 3} x + 3 = 6$$

Example 2: As $n \to \infty$

When asked to evaluate a limit as $n \to \infty$, divide above and below by the highest power of n. Then use the fact that

$$\lim_{x\to\infty} \frac{1}{x^n} = 0$$

Evaluate:

$$\lim_{x\to\infty} \frac{x^2-3x+3}{2x^2-5x-1} =$$

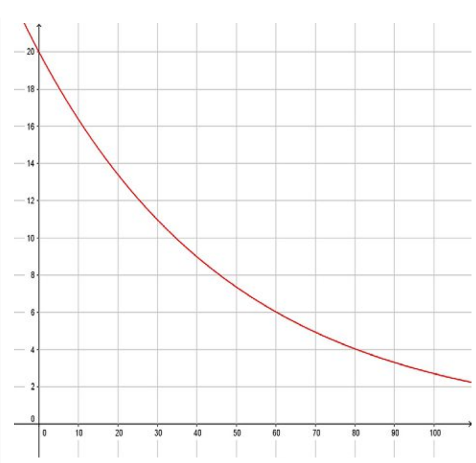
$$\lim_{x\to\infty} \frac{1-\frac{3}{x}+\frac{3}{x^2}}{2-\frac{5}{x}-\frac{1}{x^2}} = \frac{1}{2}$$

▶ Graph linear, quadratic, cubic, exponential, logarithmic and trigonometric functions and use the graphs to solve equations.

Example: Graph the function $f(x) = 20e^{(-0.02t)}$ in the domain $0 \le x \le 100$, $t \in \mathbb{R}$.

Change your calculator to table mode and insert the function with the correct domain into the calculator. This gives us the following points, rounded to one decimal place:

(0, 20), (10, 16.4), (20, 13.4), (30, 11), (40, 9), (50, 7.4), (60, 6), (70, 4.9), (80, 4), (90, 3.3), (100, 2.7)



Plotting these points, we get the above curve:

▶ Recognise surjective, injective and bijective functions. Learn the definitions of each type:

- A function is injective if for every y in the codomain, there is **at most** one x in the domain.
- A function is surjective if for every y in the codomain, there is **at least** one x in the domain.
- A function is bijective if for every y in the codomain, there is **exactly** one x in the domain.

▶ Find and sketch the inverse of a bijective function.

Example: Let $f(x) = 7x - 8$. Find an expression for $f^{(-1)}(x)$.

$$y = 7x - 8$$

$$\Rightarrow y + 8 = 7x$$

$$\Rightarrow \frac{y+8}{7} = x$$

$$\Rightarrow f^{-1}(x) = \frac{x+8}{7}$$

▶ Solve problems involving composite functions.

Example: Let

$f(x) = 3x + 1$, $g(x) = x^2 - 2$.

Is $fg(x) = gf(x)$?

Let's first find $fg(x)$:

$fg(x) = f(x^2 - 2)$
$= 3(x^2 - 2) + 1$
$= 3x^2 - 6 + 1$
$= 3x^2 - 5$

Now, find $gf(x)$:

$gf(x) = g(3x + 1)$
$= (3x + 1)^2 - 2$
$= 9x^2 + 6x + 1 - 2$
$= 9x^2 + 6x - 1$

Therefore, $fg(x) \neq gf(x)$

▶ Be familiar with continuous functions.

▶ Express quadratic functions in complete square form and use this to find the roots and turning points and sketch the function.

Example:

(a) By completing the square, find the maximum/minimum point of the function

$f(x) = 2x^2 + 3x - 2$

$$f(x) = 2\left[x^2 + \frac{3}{2}x - 1\right]$$

$$= 2\left[\left(x^2 + \frac{3}{2}x + \frac{9}{16}\right) - \frac{9}{16} - 1\right]$$

$$= 2\left[\left(x + \frac{3}{4}\right)^2 - \frac{25}{16}\right]$$

$$= 2\left(x + \frac{3}{4}\right)^2 - \frac{25}{8}$$

Since the function is U-shaped, the turning point is a minimum with coordinates

$$\left(-\frac{3}{4}, -\frac{25}{8}\right)$$

Proof by Induction

There are three types of proof by induction that can be asked. Students can also be asked to prove De Moivre's Theorem by induction. This particular proof by induction was examined in 2018.

▶ Simple identities such as:

- Prove, by induction, that the sum of the first n natural numbers,

$$1 + 2 + 3 + \cdots + n \text{ is } \frac{n(n+1)}{2}.$$

Example: Prove by induction that

$$\sum_{r=1}^{n} 5r = \frac{5n}{2}(n+1) \text{for all } n \in \mathbb{N}$$

First, write out exactly what you are being asked to prove:

To prove:

$$5 + 10 + 15 + \cdots + 5n = \frac{5n}{2}(n+1)$$

Step 1: Prove true for $n = 1$:

$$\frac{5(1)}{2}(1+1) = 5$$

Therefore the statement is true for $n = 1$.

Step 2: Assume true for $n = k$:

Assume

$$5 + 10 + 15 + \cdots + 5k = \frac{5k}{2}(k+1)$$

Step 3: Prove true for $n = k + 1$

Prove:

$$5 + 10 + 15 + \cdots + 5k + 5(k+1)$$

$$= \frac{5(k+1)}{2}(k+2)$$

Consider the left hand side:

$$5 + 10 + 15 + \cdots + 5k + 5(k+1)$$

$$= \frac{5k}{2}(k+1) + 5(k+1)$$

from the assumption.

$$= \frac{5k(k+1)}{2} + \frac{10(k+1)}{2}$$

$$= \frac{5(k+1)(k+2)}{2}$$

Step 4: Conclusion

We have proven that, if $P(k)$ is true, then $P(k+1)$ is true. But $P(1)$ is true. Therefore $P(2)$ is true and so on. Therefore, by induction, the proposition is true for all $n \in \mathbb{N}$.

▶ Simple inequalities such as:

- Prove by induction that $n! > 2^n$ for all $n \in \mathbb{N}$, $n \ge 4$
- Prove by induction that $2^n \ge n^2$ for all $n \in \mathbb{N}$, $n \ge 4$
- Prove by induction that, for $x > -1$, $(1 + x)^n \ge 1 + nx$ for all $n \in \mathbb{N}$

▶ Factorisation/Divisibility proofs

Example: Prove by induction that 3 is a factor of $4^n - 1$ for all $n \in \mathbb{N}$

Step 1: Prove true for $n = 1$:

$4^1 - 1 = 3$ is divisible by 3.

Therefore, the statement is true for $n = 1$.

Step 2: Assume true for $n = k$:

Assume

3 is a factor of $4^k - 1$ for all $k \in \mathbb{N}$

This means that:

$4^k - 1 = 3A$ for some A.

$\Rightarrow 4^k = 3A + 1$

Step 3: Prove true for $n = k + 1$

i.e. Prove that:

3 is a factor of $4^{(k+1)} - 1$

Consider $4^{(k+1)} - 1$:

$4^{(k+1)} - 1 = (4^k)(4^1) - 1$
$= 4(4^k) - 1$
$= 4(3A + 1) - 1$
(from the assumption)
$= 12A + 4 - 1$
$= 12A + 3$
which is divisible by 3.

Step 4: Conclusion

We have proven that, if $P(k)$ is true, then $P(k+1)$ is true. But $P(1)$ is true. Therefore $P(2)$ is true and so on. Therefore, by induction, the proposition is true for all $n \in \mathbb{N}$

Arithmetic & Financial Maths

- Be familiar with the following terms and how to find them: order of magnitude, accumulate error, percentage error, tolerance, cost price, selling price, loss, discount, mark up (profit as a % of cost price), margin (profit as a % of selling price).
- Be able to complete calculations involving income tax and net pay.
- Be able to calculate compound interest and depreciation (reducing balance method),
- It's very important that students can change interest rates, depending on the time intervals.

Example: Find the AER that is equivalent to a monthly interest rate of 0.6279%

Let i = annual interest rate.

$$1.006279 = (1+i)^{\frac{1}{12}}$$
$$1.006279^{12} = 1+i$$
$$i = 1.006279^{12} - 1$$
$$i = 0.078$$

Therefore, the annual rate is 7.8%

- You should be able to use the sum of a geometric series to complete calculations involving present and future values. Remember to use future values when investing money and present values for loans.

Example 1: €1,000 is invested into an account at the beginning of each year for 5 years at an AER of 3%. How much is in the account after 5 years?

This is a geometric series:

$$1000\,(1.03) + 1000\,(1.03)^2 + 1000\,(1.03)^3 + 1000\,(1.03)^4 + 1000\,(1.03)^5$$

Using the formula for the sum of a geometric series, we get:

$$\frac{1000(1.03)(1.03^5 - 1)}{1.03 - 1} = €5{,}468.41$$

Example 2: In order to buy a car, I take a loan of €10,000 at a monthly interest rate of 0.75%. How much is each monthly repayment, if I want to make 24 equal payments?

We can use the amortisation formula to answer this question:

$$A = P\frac{i(1+i)^t}{(1+i)^t - 1}$$

$$= (10{,}000)\frac{(0.0075)(1.0075)^{24}}{(1.0075)^{24} - 1}$$

$$= €456.85$$

- Be able to derive the formula for a mortgage repayment.

Calculus

It is really important that students understand what differentiation is. Always remember that a derivative is a rate of change. It is also the slope of the tangent to the curve.

- You need to know how to differentiate linear and quadratic functions from first principles.
- The definition required for differentiation from first principles must be **learnt off**:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Example: Differentiate the function $f(x) = x^2$ from first principles.

$$f(x) = x^2$$
$$\Rightarrow f(x+h) = (x+h)^2$$
$$= x^2 + 2xh + h^2$$
$$\Rightarrow f(x+h) - f(x)$$
$$= (x^2 + 2xh + h^2) - (x^2)$$
$$= 2xh + h^2$$
$$\Rightarrow \frac{f(x+h) - f(x)}{h} = \frac{2xh + h^2}{h}$$
$$= 2x + h$$
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h \to 0} 2x + h$$
$$= 2x$$

- Be able to differentiate various functions and use the product, quotient and chain rules.

Example: Find the derivative of the function

$$y = x^2 \sin(3x)$$

Since this is a product of two functions, we will be using the product rule.

Let $u = x^2$

$$\Rightarrow \frac{du}{dx} = 2x$$

Let $v = \sin 3x$

(We need to use the chain rule here.)

$$\Rightarrow \frac{dv}{dx} = 3\cos 3x$$

Then, using the product rule, we find that:

$$\frac{dy}{dx} =$$
$$(x^2)(3\cos 3x) + (\sin 3x)(2x)$$
$$= 3x^2 \cos 3x + 2x \sin 3x$$

- Be able to deal with rates of change. Remember to use the area and volume formulae where necessary. Don't forget to include units in your answer.

Example: A spherical balloon is being inflated at a rate of $50\text{cm}^3/s$ Find the rate at which the radius is increasing when the radius of the balloon is 10cm.

$$V = \frac{4}{3}\pi r^3$$

$$\Rightarrow \frac{dV}{dr} = 4\pi r^2$$

From the question, we know that:

$$\frac{dV}{dt} = 50$$

Set up a chain rule:

$$\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$$
$$= \frac{1}{4\pi r^2} \times 50$$
$$= \frac{25}{2\pi r^2}$$

When $r = 10$,

$$\frac{dr}{dt} = \frac{25}{2\pi r^2} = \frac{1}{8\pi}\, cm/s$$

- Be able to find the maxima and minima of curves. Use differentiation to sketch curves using turning points, intercepts, points of inflection, increasing and decreasing curves.

Example: Let

$$f(x) = 2x^2 - 3x + 2$$

for $x \in \mathbb{R}$. Find the co-ordinates of the points at which the slope of the tangent to the curve $y = f(x)$ is 5.

$f'(x)$ = the slope of the tangent to the curve.

Therefore, $f'(x) = 5$

Therefore, $f'(x) = 4x - 3 = 5$

$$4x = 8 \Rightarrow x = 2$$

To find the y co-ordinate, find $f(2)$:

$$f(2) = 2(2)^2 - 3(2) + 2 = 2$$

Therefore, the point is (2, 2).

- Use differentiation to solve problems involving maxima and minima.

Example: Find the dimensions of the rectangle of largest area having fixed perimeter 100cm.

We want to maximise the area of the rectangle. Let l represent the length and w represent the width of the rectangle. Therefore

$$A = lw$$

In order to write A in terms of one variable, we use the information about the perimeter, P.

$$P = 2l + 2w = 100$$
$$l + w = 50$$
$$w = 50 - l$$

Therefore $A = l(50 - l)$

$$A = 50l - l^2$$
$$\frac{dA}{dl} = 50 - 2l$$

At a maximum,

$$\frac{dA}{dl} = 0$$

Therefore, $50 - 2l = 0$

$$l = 25\text{cm}$$

And $w = 25$cm

- Be able to sketch the slope function of a curve.
- Be able to find the slope of a tangent to a circle. This was added to the syllabus in recent years and **has never been examined**. It can be done using implicit differentiation.

Example: Find the slope of the tangent to the circle

$$x^2 + y^2 - 2x - 2y - 23 = 0 \text{ at the point } (4, -3).$$

$$x^2 + y^2 - 2x - 2y - 23 = 0$$
$$2x + 2y\frac{dy}{dx} - 2 - 2\frac{dy}{dx} = 0$$
$$(2y - 2)\frac{dy}{dx} = -2x + 2$$
$$\frac{dy}{dx} = \frac{-2x + 2}{2y - 2}$$

At the point $(4, -3)$,

$$\frac{dy}{dx} = \frac{3}{4}$$

Therefore, the slope of the tangent to the circle at the point $(3, 4)$ is $\frac{3}{4}$.

- Integrate various functions

Example: Evaluate the integral

$$\int_0^{\frac{\pi}{4}} \cos 2x\, dx$$

$$\int_0^{\frac{\pi}{4}} \cos 2x\, dx = \left[\frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{4}}$$
$$= \frac{1}{2}\sin\frac{2\pi}{4} - \frac{1}{2}\sin 2(0)$$
$$= \frac{1}{2}(1) - 0 = \frac{1}{2}$$

- Use integration to find the average value of a function over an interval. **Learn** this formula!

The average value of a function $y = f(x)$ over an interval $[a, b]$ is:

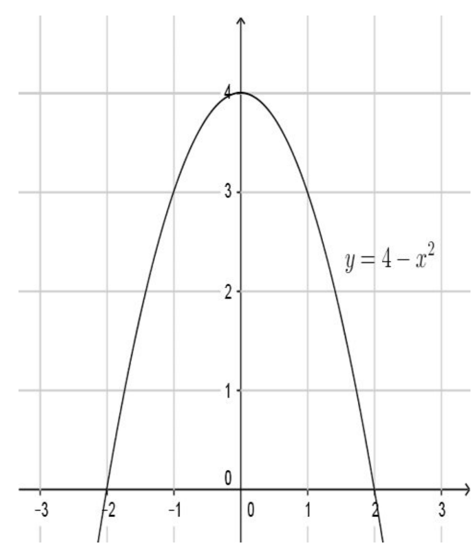
$$\frac{1}{b-a}\int_a^b y\, dx$$

Example: Find the average value of the function $f(x) = 2x^2 - 3x + 2$ on the interval $[0, 2]$.

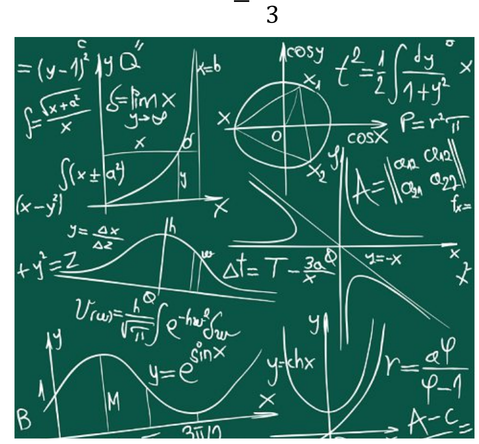
$$= \frac{1}{2-0}\int_0^2 (2x^2 - 3x + 2)\, dx$$
$$= \frac{1}{2}\left[\frac{2x^3}{3} - \frac{3x^2}{2} + 2x\right]_0^2$$
$$= \frac{1}{2}\left[\frac{2(2)^3}{3} - \frac{3(2)^2}{2} + 2(2)\right]$$
$$= \frac{1}{2}\left(\frac{10}{3}\right)$$
$$= \frac{5}{3}$$

- Determine areas of plane regions bounded by polynomial and exponential curves

Example: Find the area between the curve $y = 4 - x^2$ and the X-axis, as is shown in the diagram



$$\int_{-2}^{2} (4 - x^2)dx =$$
$$\left[4x - \frac{x^3}{3}\right]_{-2}^{2}$$
$$= \left[(4)(2) - \frac{(2)^3}{3}\right]$$
$$-\left[(4)(-2) - \frac{(-2)^3}{3}\right]$$
$$= \left[8 - \frac{8}{3}\right] - \left[-8 + \frac{8}{3}\right]$$
$$= 8 - \frac{8}{3} + 8 - \frac{8}{3}$$
$$= \frac{32}{3}$$



Paper Two

Learning

Make sure you learning the following for Paper 2.

- **Definitions of the following types of samples:** simple random, cluster, stratified, systematic, quota, convenience.

- **Students need to be able to make decisions based on the Empirical Rule:** In a normal distribution:
  → 68% of the data lies within one standard deviation of the mean.
  → 95% of the data lies within two standard deviations of the mean.
  → 99.7% of the data lies within three standard deviations of the mean.

- **The Central Limit Theorem:** If a population is normally distributed, then the sampling distribution will also be normal.
  For **any** population distribution, the sampling distribution will be approximately normal for sample size $n \geq 30$

- **Learn the rule for *p* values:** If $p < 0.05$, the null hypothesis is rejected. If $p \geq 0.05$, do not reject the null hypothesis.

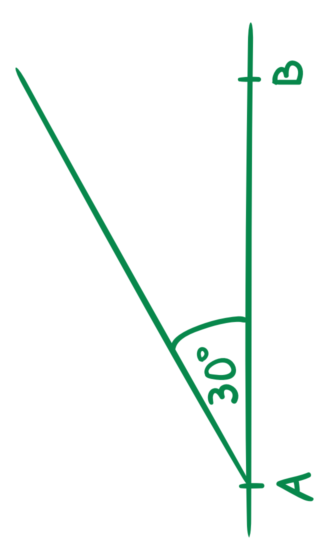
- For two events A and B:
  → A and B are mutually exclusive if

$$A \cap B = \emptyset$$

  → $P(A \text{ or } B) =$

$$P(A) + P(B) - P(A \cap B)$$

  → A and B are independent if the occurrence of event A does not change the probability of event B.
  → If A and B are independent, then $P(A \text{ and } B) = P(A)\,P(B)$
  → If A and B are not independent, then $P(A \text{ and } B) = P(A)\,P(B|A)$
  → $P(A|B) = \frac{P(A \cap B)}{P(B)}$



- Learn the formula for finding the expected value:
  $E(x) = xP(x)$

- Learn the proofs of theorems 11, 12 and 13.

- Be able to complete all the constructions. Pay particular attention to the centres of the triangle.

- Learn the following terms related to logic and deductive reasoning: theorem, proof, axiom, corollary, converse, implies, equivalent to, if and only if, proof by contradiction.

- Learn to derive the trigonometric formulae 1, 2, 3, 4, 5, 6, 7, 9.

- Learn the formulae for volume and area of a rectangular block and a hemisphere as these are not in the formulae tables.

Statistics

Statistics and Probability make up a considerable amount of Paper 2. Questions on these two topics are often linked and appear in both sections of the paper. Students are expected to be able to draw conclusions based on statistics. Be precise with your responses and use the correct terminology.

- Students need to be able to find, collect and organise data. Understand how to collect unbiased data. Be able to describe a sample and how to collect one.

- Students need to be able to represent data using suitable graphs. Be aware of the types of graphs and when to use them. Use a histogram to represent continuous data. And use a scatter plot to graph bivariate data. You need to be able to comment on distribution of data.

- Students should also be able to determine the relationship between variables using scatter plots and analyse these using correlation coefficients and a line of best fit. When describing correlation between two variables, comment on both the direction and strength of the correlation. Always remember to interpret your data in the context of the question asked.

Example: Find the correlation coefficient for the following data. Describe and interpret the correlation.

Height (cm)	170	175	183	162	177
Weight (kg)	66	70	75	65	72

Use your calculator in STAT mode to find the correlation coefficient r:

$$r = 0.95797$$

In conclusion, there is a strong positive correlation between the height and weight of the people asked. As height increases, weight also increases.

- Students need to be able to analyse data numerically by calculating mean, mode, median, standard deviation, interquartile range and percentiles. You also need to be able to refer to outliers.
- Students need to be able to make decisions based on the Empirical Rule.

Example: Research was performed on the IQ scores of the employees of a private firm. The scores are noted to be in normal distribution. The mean of the distribution was 100 and standard deviation was 15. Estimate the percentage of the scores that fall between 70 and 130.

95% of the data lies within two standard deviations of the mean. Therefore,

$$P(70 \le x \le 130) = 0.95$$

- Calculate z-scores in a normal distribution.

Example: In a Maths test, the mean result was 62% and the standard deviation was 12. Calculate the z-score for a test that scored 74%. What does your result mean?

$$z = \frac{x-\mu}{\sigma}$$

$$z = \frac{74-62}{12} = 1$$

$z = 1$ means that the result is one standard deviation above the mean.

- **Inferential Statistics** has appeared as a question every year since 2015. It is mostly a Section B question. Although there is no guarantee that this topic will appear on the paper, students should have it well prepared. Students need to understand the terms population, sample, population/sample proportion, population/sample mean. Be clear on the notation:
  μ means population mean
  $\bar{x}$ means sample mean
  p means population proportion
  $\hat{p}$ means sample proportion.

- Students need to be able to calculate probabilities involving sample means and sample proportions.

Example: A random sample of size 36 is chosen from a population with mean 12 and standard deviation 3. Find the probability that the sample mean is greater than 13.

Since $n = 36$, the sampling distribution of the mean is approximately normal with mean 12 and standard deviation

$$\frac{3}{\sqrt{36}} = \frac{1}{2}$$

Convert to z-score:

$$z = \frac{13-12}{\frac{1}{2}} = 2$$

Therefore $P(x > 13) = P(z > 2) =$
$1 - P(z < 2) =$
$1 - 0.9772 = 0.0228$

Example: A population proportion is 0.45. A random sample of 50 is chosen. What is the probability that the sample proportion is less than 0.42?

$$P(\hat{p} < 0.42) =$$

$$P\left(z < \frac{0.42-0.45}{\sqrt{\frac{(0.45)(0.55)}{50}}}\right)$$

$$= P(z < -0.43)$$

$$= 1 - 0.6664 = 0.3336$$

- Students also need to be familiar with the Central Limit Theorem. Remember that, if the population is not normally distributed, sample sizes with $n \ge 30$ must be used.

- Use the margin of error formula $\frac{1}{\sqrt{n}}$ to work out the sample size that would be required in order to estimate a population proportion to a certain accuracy.

Example: A statistician wants to conduct a survey but wants to be accurate to within 5% of the population proportion. Find the minimum sample size required.

$$\frac{1}{\sqrt{n}} = 0.05$$

$$\sqrt{n} = \frac{1}{0.05}$$

$$n = \left(\frac{1}{0.05}\right)^2$$

$$n = 400$$

- Construct 95% confidence intervals for the population mean from a large sample **and** for the population proportion, in both cases using z tables. Practice using the **appropriate formulae** from page 34 of the *Formulae and Tables Booklet*.

Example: In a random sample of 50 students taking an exam, it was found that the mean mark was 48.6 with standard deviation 8.5. Calculate a 95% confidence interval for the mean score of all students who took the exam.

$$\bar{x} = 48.6,$$

$$\frac{\sigma}{\sqrt{n}} = \frac{8.5}{\sqrt{50}}$$

Therefore

$$48.6 - 1.96\left(\frac{8.5}{\sqrt{50}}\right) < \mu < 48.6 + 1.96\left(\frac{8.5}{\sqrt{50}}\right)$$

$$46.24 < \mu < 50.96$$

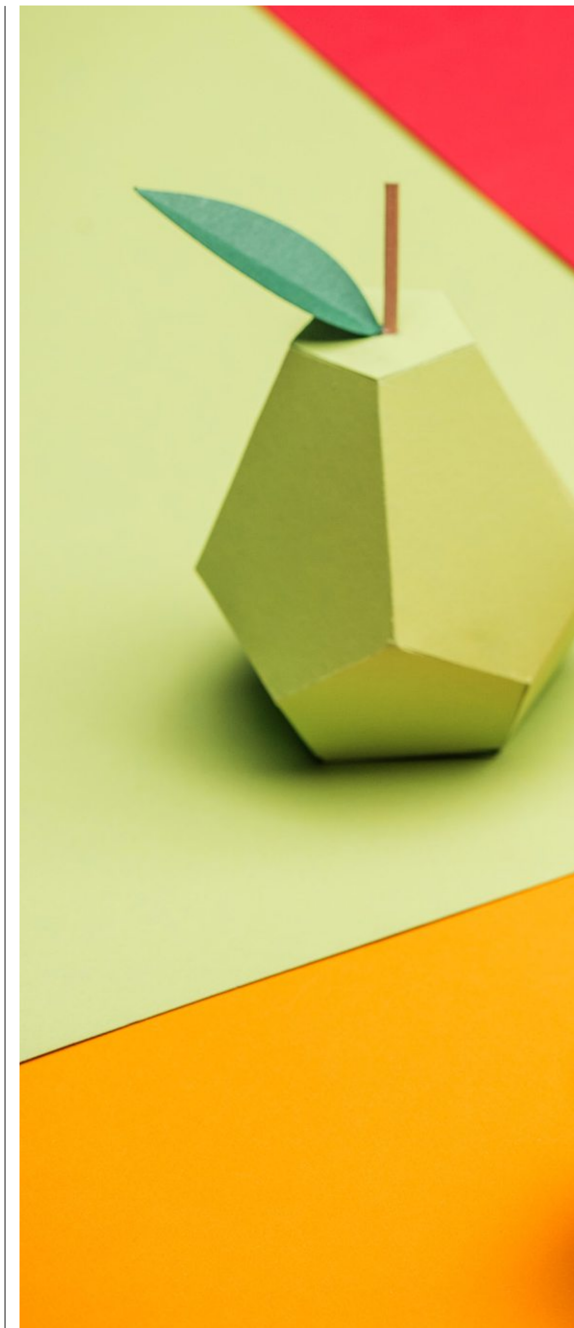
- Conduct a hypothesis test on a population proportion and a population mean.

- Use and interpret p-values. If $p < 0.05$, the null hypothesis is rejected. If $p \ge 0.05$, do not reject the null hypothesis. To remember this, it's helpful to learn the phrase: "If p is low, H_0 must go".

Example: Calculate the p-value for a z-score of 2.25.

p-value $= 2P(z > 2.25)$
$2P(z>2.25) =$
$2[1 - P(z < 2.25)]$
$= 2[1 - 0.9878]$
$= 2(0.0122)$
$= 0.0244$

Since the p-value ≤ 0.05, we reject the null hypothesis.



Probability

Students need to know how to count the number of ways of selecting and arranging objects. They need to be familiar with the notation of choosing and factorials and when to use which method.

Example: A school committee of four has to be chosen from six girls and seven boys. How many different committees can be chosen if the team must have two girls and two boys?

We need to choose two girls from a possible six and we need to choose two boys from a possible seven. The amount of possible combinations is:

$$\binom{6}{2} \times \binom{7}{2} = 315$$

- Be able to solve problems using the rules of probability. Understand the rules using Venn diagrams and formulae.

Example: A bag contains 6 red marbles and 4 blue marbles. Two marbles are chosen at random and not replaced. Find the probability that they are different colours.

$$P(\text{different colour}) = P(RB \text{ or } BR)$$

$$= \left(\frac{6}{10} \times \frac{4}{9}\right) + \left(\frac{4}{10} \times \frac{6}{9}\right)$$

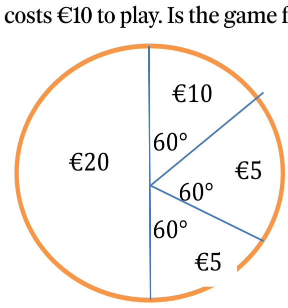
$$= \left(\frac{4}{15}\right) + \left(\frac{4}{15}\right)$$

$$= \frac{8}{15}$$

- Solve problems involving expected value. **Learn** the formula below. This concept is examined regularly.

$$E(x) = xP(x)$$

Example: In the given wheel, you win the amount written on the sector in which the arrow stops. It costs €10 to play. Is the game fair?



x	$P(x)$
€5	$\frac{1}{3}$
€10	$\frac{1}{6}$
€20	$\frac{1}{2}$

$$E(X) = (5)\left(\frac{1}{3}\right) + (10)\left(\frac{1}{6}\right) + (20)\left(\frac{1}{2}\right)$$
$$= 13\frac{1}{3}$$

It costs €10 to play so you expect to win €3.33

▶ Solve problems using Bernoulli trials.

Example 1: A fair die is rolled 3 times. Find the probability of getting 2 fours.

The probability of getting 2 fours is:

$$= \binom{3}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^1$$
$$= \frac{5}{72}$$

Example 2: A dice is rolled 6 times. What is the probability of rolling at least 1 one?

P(*at least one*) = 1 — P(*none*)

Therefore,

P(*at least one*)

$$= 1 - \left[\binom{6}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^6\right]$$
$$= 1 - 0.3349$$
$$= 0.6651$$

▶ Solve problems involving conditional probability

Example: In a school there are 150 girls and 100 boys. 120 of the girls play hockey and 50 boys play hockey. A student is chosen at random. What is the probability that the student is a girl, given that they play hockey?

$$P(G|H) = \frac{P(G \cap H)}{P(H)}$$
$$= \frac{\frac{120}{250}}{\frac{170}{250}} = \frac{12}{17}$$

▶ Solve problems involving independent events. Remember: If A and B are independent, then

$P(A \cap B) = P(A)P(B)$

Example: Two events A and B are such that $P(A) = 0.4$, $P(B) = 0.5$ and $P(A \cup B) = 0.8$. Are A and B independent events?

In order to check if A and B are independent events, we first need to find $P(A \cap B)$.

In order to do this,we can use the formula:

$P(A \cup B) = P(A) + P(B) — P(A \cap B)$

$0.8 = 0.4 + 0.5 — P(A \cap B)$

$P(A \cap B) = 0.1$

But, $P(A)\,P(B) = 0.4 \times 0.5 = 0.2$

$P(A \cap B) \neq P(A)\,P(B)$

Therefore, A and B are not independent events.

▶ Solve problems involving reading probabilities from the normal distribution tables. Remember if the question says that the data is normally disturbed, you should be using z-scores.

Example: A random variable X has a normal distribution with mean 10 and standard deviation 2.

Find $P(8 \le X \le 12)$.

$P(8 \le X \le 12)$ =

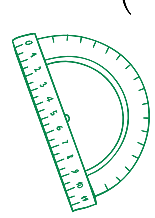
$$P\left(\frac{8-10}{2} \le z \le \frac{12-10}{2}\right)$$

$= P(-1 \le z \le 1)$

$= 2P(z \le 1) - 1$

$= 2\,(0.8413) - 1$

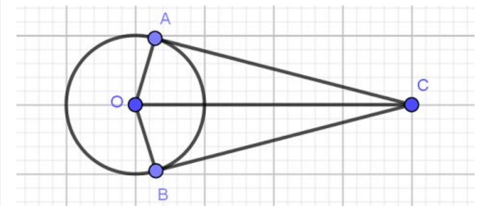
$= 0.6826$



Geometry

Students need to be familiar with all theorems on the course and how to apply them to solve problems. If asked to prove something, present your work clearly and logically. Always write a conclusion! The theorems involve facts about lines, angles, triangles, quadrilaterals and circles.

Example: AC and BC are tangents to the circle shown with centre O. Prove that the triangles AOC and BOC are congruent.



$|AO| = |OB|$ ...... both radii

$|\angle OAC| = |\angle OBC| = 90°$ ..... tangents

$|OC| = |OC|$ ...... common

Therefore, $\Delta AOC \equiv \Delta BOC$ ...... RHS

▶ Learn the proofs of theorems 11, 12 and 13.

▶ Learn all the constructions. Students need to understand the process involved. They can be asked to use these to solve problems. Pay particular attention to the following details involving the centres of the triangles and their properties.

● The **circumcentre** of a triangle is the point where the perpendicular bisectors of the sides intersect. The circumcircle is the circle that passes through all three vertices of the triangle. The circumcentre is **inside** all acute triangles; **outside** all obtuse triangles and **on** all right triangles (at the midpoint of the hypotenuse)

● The **incentre** is the point of intersection of the three angle bisectors of any triangle. The incentre is always inside the triangle.

● The **centroid** of a triangle is the point where its medians intersect. A median is a line drawn from one vertex of a triangle to the midpoint of the opposite side. The centroid is also known as the centre of gravity. It is always inside the triangle. The centroid divides each median in the ratio 2:1.

● The **orthocentre** is the point where all three altitudes of the triangle intersect. An altitude is a line which passes through a vertex of the triangle and is perpendicular to the opposite side. The orthocentre is **inside** all acute triangles; **outside** all obtuse triangles and **on** all right triangles (on the vertex at the right angle).

For example, if asked to construct the location of a fire station that is equidistant from each of three towns, construct the circumcentre.

If asked to construct three roads equidistant from a central point, use the incentre and if asked for the centre of gravity, construct the centroid.

▶ Learn the following terms related to logic and deductive reasoning: theorem, proof, axiom, corollary, converse, implies, equivalent to, if and only if, proof by contradiction.

▶ Students need to be able to solve problems involving enlargements. Learn the properties of enlargements and their construction.

Coordinate Geometry

Students need to be able to use the formulae for coordinate geometry of the line including the distance and midpoint formulae.

- Solve problems involving slopes of lines. Make sure you have a full understanding of the different ways to get slope. Depending on the information given, the method to find slope changes:
  - If a line is in the form $ax + by + c = 0$ its slope is $-\frac{a}{b}$.
  - If a line is in the form $y = mx + c$, its slope is m.
  - If given coordinates of two points, use the formula to find the slope of the line.
  - If given a graph, find slope using $\frac{rise}{run}$
  - Remember that $\tan\theta$ equals slope where θ is the angle that the line makes with the positive direction of the X-axis.
  - **Learn:** If l_1 and l_2 have slopes m_1 and m_2 respectively then:

$$l_1 \parallel l_2 \Leftrightarrow m_1 = m_2$$
$$l_1 \perp l_2 \Leftrightarrow m_1 \times m_2 = -1$$

Example: The line AB is perpendicular to the line CD where $A(-1, 5)$, $B(3, -3)$, $C(10, 3)$, $D(x, 0)$. Find x.

$$\text{Slope of } AB = \frac{-3-5}{3+1} = -2$$

$$CD \perp AB \Rightarrow \text{slope } CD = \frac{1}{2}$$

$$\Rightarrow \frac{0-3}{x-10} = \frac{1}{2}$$

$$\Rightarrow 2(-3) = x - 10$$

$$\Rightarrow x = 4$$

- Find the equation of a line.

- Find the area of a triangle. Remember: when using the formula for the area of a triangle, one of the vertices must be at (0, 0).

Example: Find the area of the triangle ABC where $A(2, 1)$, $B(3, 8)$ and $C(-2, 5)$.

We first move one of the points to the origin and move the others by the same translation:

$(2, 1) \to (0, 0)$
$(3, 8) \to (1, 7)$
$(-2, 5) \to (-4, 4)$

We can now use the formula:

$$\frac{1}{2}|x_1y_2 - x_2y_1|$$

$$= \frac{1}{2}|(1)(4) - (7)(-4)|$$

$$= \frac{1}{2}|32|$$

$$= 16\ units^2$$

- Use the formula to find the perpendicular distance from a point to a line.

Example: Find the distance between the pair of parallel lines $2x - y - 2 = 0$ and $2x - y + 5 = 0$.

To solve this, we find a point on either line then use the formula to find the distance to the other line.

Consider the line $2x - y - 2 = 0$. The point $(0, -2)$ is on this line. We need to find the distance from the point $(0, -2)$ to the line $2x - y + 5 = 0$. Label the point $(0, -2)$ as (x_1, y_1) and from the equation of the line, we get that $a = 2$, $b = -1$, $c = 5$. Subbing these values into the formula we get that the distance equals:

$$\frac{|(2)(0) + (-1)(-2) + 5|}{\sqrt{(2)^2 + (-1)^2}} = \frac{7\sqrt{5}}{5}$$

- Solve problems involving the angle between two lines. We only use the ± sign when using the formula to find the slope of a line. If you're using the formula to find the angle, find the positive value to get the acute angle, then the obtuse angle is $180 - \theta$.

Example: Find the slopes of the lines that make an angle of 45º with the line $3x + 2y - 4 = 0$.

The slope we are trying to find, we call m. The slope of $3x + 2y - 4 = 0$ is $-\frac{3}{2}$. We sub the following values into our formula: $\theta = 45, m_1 = m, m_2 = -\frac{3}{2}$

$$\tan 45 = \pm\left(\frac{m - \left(-\frac{3}{2}\right)}{1 + (m)\left(-\frac{3}{2}\right)}\right)$$

$$1 = \pm\left(\frac{m + \frac{3}{2}}{1 - \frac{3m}{2}}\right)$$

$$1 - \frac{3m}{2} = \pm\left(m + \frac{3}{2}\right)$$

$$2 - 3m = \pm(2m + 3)$$

First we solve:

$$2 - 3m = +(2m + 3)$$
$$5m = -1$$
$$m = -\frac{1}{5}$$

Next, we consider the case:

$$2 - 3m = -(2m + 3)$$
$$2 - 3m = -2m - 3$$
$$m = 5$$

- Students need to be able to divide a line segment internally in a given ratio m:n.

- Students need to be familiar with the equations of the circle and find the centre and radius from an equation. Students need to be able to find the equation of a circle given certain information. When asked to find the equation of a circle, students should draw a diagram to help visualise the problem.

When trying to find the equation of a circle, let the equation be

$$x^2 + y^2 + 2gx + 2fy + c = 0,$$

then find three equations in g, f and c and solve for them.

Example: Find the equations of the circles that contains the point $(3, -1)$, has radius 5 and whose centre is on the line

$$2x + 3y = 4.$$

Let the circle have equation:

$$s: x^2 + y^2 + 2gx + 2fy + c = 0$$

$(3, -1)$ is on s so sub it in:

$$(3)^2 + (-1)^2 + 2g(3) + 2f(-1) + c = 0$$
$$\Rightarrow 6g - 2f + c = -10$$

The centre $(-g, -f)$ is on the line

$$2x + 3y = 4 \Rightarrow -2g - 3f = 4$$

The radius is 5:

$$\sqrt{g^2 + f^2 - c} = 5$$
$$g^2 + f^2 - c = 25$$
$$-6g - c = 9$$

Rearrange the equations:

Rearrange $-2g - 3f = 4$ to get $g = -2 - \frac{3f}{2}$

Rearrange $6g - 2f + c = -10$ to get $c = -6g + 2f - 10$

Therefore $c = -6(-2 - \frac{3f}{2}) + 2f - 10$

$$\Rightarrow c = 11f + 2$$

Sub both of these into

$$g^2 + f^2 - c = 25$$

$$\left(-2 - \frac{3f}{2}\right)^2 + f^2 - (11f + 2) = 25$$

$$13f^2 - 20f - 92 = 0$$

$$(13f - 46)(f + 2) = 0$$

$$f = \frac{46}{13}, -2$$

Therefore, $g = -\frac{95}{13}, 1$

And $c = \frac{532}{13}, -20$

Therefore the circles have equations

$$13x^2 + 13y^2 - 190x + 92y + 532 = 0$$

and

$$x^2 + y^2 + 2x - 4y - 20 = 0$$

- Solve problems involving a line and a circle. Students need to know how to get the point(s) of intersection of a line and a circle. It is important to be able to solve problems involving tangents.

Example: Find the slopes of the tangents from the point $(7, 1)$ to the circle $x^2 + y^2 = 25$

From the question, it is clear that the point $(7, 1)$ is not on the circle. It is a point outside the circle so we need to use the perpendicular distance formula to find the slope of the tangent.

Let m be the slope of the tangent.

Therefore, the equation of the tangent is

$$y - 1 = m(x - 7)$$
$$y - 1 = mx - 7m$$
$$mx - y + 1 - 7m = 0$$

The perpendicular distance from the centre of the circle to the tangent equals the radius. (This concept is used frequently to solve circle questions).

Using the perpendicular distance formula with $(x_1, y_1) = (0, 0)$ and $a = m$, $b = -1$, $c = 1 - 7m$, we get:

$$\frac{|(0)(m) + (0)(-1) + 1 - 7m|}{\sqrt{m^2 + (-1)^2}} = 5$$

$$|1 - 7m| = 5\sqrt{m^2 + 1}$$

Square both sides:

$$1 - 14m + 49m^2 = 25(m^2 + 1)$$

$$1 - 14m + 49m^2 = 25m^2 + 25$$

$$24m^2 - 14m - 24 = 0$$
$$12m^2 - 7m - 12 = 0$$
$$(3m - 4)(4m + 3) = 0$$

$$m = \frac{4}{3}, -\frac{3}{4}$$

- Solve problems involving intersecting circles. Remember that, if a circle S_1 has centre c_1 and radius r_1 and another circle S_2 has centre c_2 and radius r_2 then:
  - the **circles touch externally** if the distance between their centres equals the sum of their radii: $|c_1 c_2| = r_1 + r_2$
  - the **circles touch internally** if the distance between their centres equals the difference of their radii: $|c_1 c_2| = |r_1 - r_2|$

Example: The circle s_1 has equation $x^2 + y^2 - 6x + 4y - 12 = 0$.

s_2: $x^2 + y^2 + 12x - 20y + k = 0$ is another circle. If the two circles touch externally, find the value of k.

Firstly, we need to find the centre and radius of s_1: centre $= c_1 = (3, -2)$ and radius $= r_1 =$

$$\sqrt{(-3)^2 + (2)^2 + 12} = 5.$$

The circle s_2 has centre $c_2 = (-6, 10)$ and radius $= r_2 =$

$$\sqrt{(6)^2 + (-10)^2 - k} = \sqrt{136 - k}.$$

Since the circles touch externally: $|c_1 c_2| = r_1 + r_2$

We use the distance formula to find $|c_1 c_2|$:

$$|c_1c_2| = \sqrt{(-6-3)^2 + (10+2)^2}$$
$$= 15$$

Therefore:

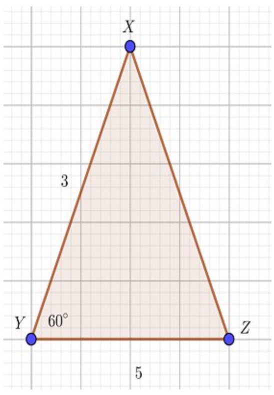
$$15 = 5 + \sqrt{136 - k}$$
$$10 = \sqrt{136 - k}$$
$$100 = 136 - k$$
$$k = 36$$

Trigonometry

Trigonometry is a really important topic. In the past, it has appeared in both sections of the paper. Students need to master the skills involved in all aspects of this topic. There are four main subtopics: solving triangles, trigonometric equations, identities (including proofs) and functions.

- Students need to be able to solve triangles. Be familiar with the formulae in the formulae tables. Practice how to find sides, angles and area. Make sure to know which formulae are used for right-angled triangles and which ones are used for other triangles. Students can be asked to solve 2D or 3D problems.

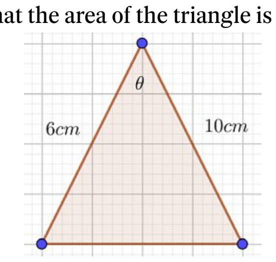
Example 1: In a triangle XYZ, $|XY| = 3$cm, $|YZ| = 5$cm and $|\angle XYZ| = 60°$. Find $|XZ|$.



Using the Cosine Rule:

$$x^2 = (3)^2 + (5)^2 - 2(3)(5)\cos 60$$
$$x^2 = 19$$
$$x = \sqrt{19}$$

Example 2: Find two values of θ given that the area of the triangle is $15cm^2$



The area of a triangle is found using

$$\frac{1}{2}ab\sin C$$

Therefore,

$$\frac{1}{2}(6)(10)\sin\theta = 15$$
$$30\sin\theta = 15$$
$$\sin\theta = \frac{1}{2}$$
$$\theta = 30°, 150°$$

▶ Students need to be able to define $\sin\theta$, $\cos\theta$ and $\tan\theta$ for all values of θ. Also solve trigonometric equations such as $\sin n\theta = 0$ and $\cos n\theta = ½$ giving all solutions.

Example 1: Solve the equation

$\sin 2x = -\frac{1}{2}$ for $0 \leq x \leq 360°$

If $0 \leq x \leq 360°$ then $0 \leq 2x \leq 720°$

Find the reference angle:

$\sin^{-1}\left(\frac{1}{2}\right) = 30°$

Find where $\sin x$ is negative: 3rd and 4th quadrants. Therefore,

$2x = (180 + 30)° = 210°$ and $(210 + 360) = 570°$.

Also $2x = (360 - 30)° = 330°$,

and $(330 + 360) = 690°$

Dividing by 2, we find that

$$x = 105°, 165°, 285°, 345°$$

Example 2: Solve the equation

$$4\cos^2 x + 4\cos x + 1 = 0$$

for $0 \leq x \leq 360°$

This is a quadratic equation, so we first need to factorise it:

$$4\cos^2 x + 4\cos x + 1 = 0$$
$$(2\cos x + 1)(2\cos x + 1) = 0$$

Therefore,

$$\cos x = -\frac{1}{2}$$

Find the reference angle:

$\cos^{-1}\left(\frac{1}{2}\right) = 60°$

Find where $\cos\ x$ is negative:

2nd and 3rd quadrants.

Therefore,

$$x = 180 - 60 = 120$$

and $x = 180 + 60 = 240$

▶ Solve problems involving the area of a sector of a circle and the length of an arc. Be able to use the relevant formulae.

▶ Graph the trigonometric functions and be able to solve problems involving the graphs. Learn the general rule:
The functions
$y = a + b\sin cx$ and $y = a + b\cos cx$ have:

$$\text{Range} = [a - b, a + b]$$

$$\text{Period} = \frac{360}{c} \text{ or } \frac{2\pi}{c}$$

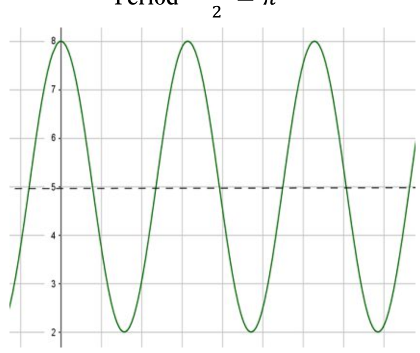
Example 1: Write down the period and range of the function

$f(x) = 5 + 3\cos 2x$

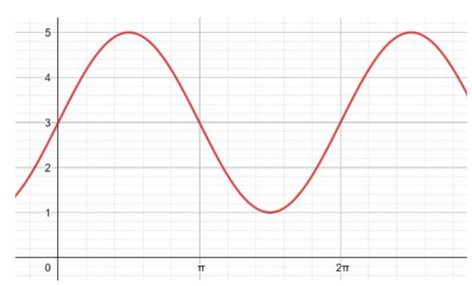
and graph the function.

Range = $[5 - 3, 5 + 3] = [2, 8]$

Period = $\frac{2\pi}{2} = \pi$



Example 2: Write down the period and range of the following functions and find an expression for $f(x)$.



From the graph of the function, we can see that:

Range = $[1, 5]$

Period = 2π

We can now use the period and range to find an expression for $f(x)$:

Range = $[1, 5] = [a - b, a + b]$

Solving this, we find that

$a = 3, b = 2$

Period $= 2\pi = \frac{2\pi}{1}$

Therefore, $c = 1$.

Therefore,

$f(x) = 3 + 2\sin x$

▶ Be able to derive the trigonometric formulae 1, 2, 3, 4, 5, 6, 7, 9.

Students also need to be able to apply the trigonometric formulae 1-24.

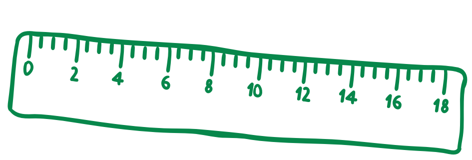
Example:

Prove that

$$\sin^4\theta - \cos^4\theta = \cos 2\theta$$

Proof

$$\sin^4\theta - \cos^4\theta$$
$$= (\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)$$
$$= (1)(\cos 2\theta)$$
$$= \cos 2\theta$$



Length, Area and Volume

This topic is a Paper 2 topic. However, students need to be familiar with it for Paper 1 because it can be required for calculus questions. Students need to be aware that not all the relevant formulae are in the *Formulae and Tables Booklet*.

▶ Students need to be able to investigate the nets of prisms, cylinders and cones.

▶ Solve problems involving the length of the perimeter and the area of plane figures: disc, triangle, rectangle, square, parallelogram, trapezium, sectors of discs, and combinations of these.

▶ Solve problems involving surface area and volume of the following solid figures: rectangular block, cylinder, right cone, triangular-based prism (right angle, isosceles and equilateral), sphere, hemisphere, and combinations of these. **Learn** the formulae for a rectangular block and a hemisphere as these are not in the *Formulae and Tables Booklet*.

Example: A sphere of diameter 6cm is dropped into a cylindrical vessel partly filled with water. The diameter of the cylindrical vessel is 12cm. If the sphere is completely submerged in water,by how much will the level of water rise in the cylindrical vessel?

The volume of the sphere is:

$$\frac{4}{3}\pi(3)^3 = 36\pi cm^3$$

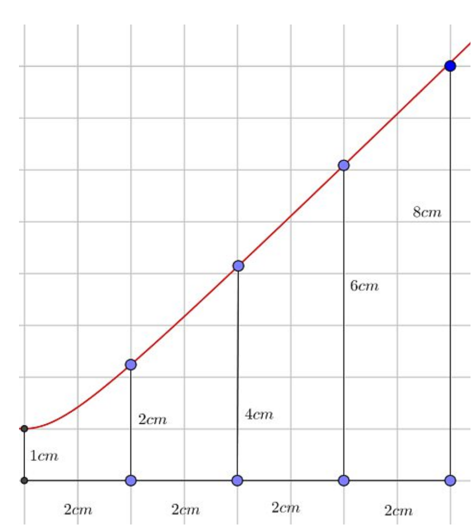
The volume of the sphere is equal to the rise in water. Let h denote the height of the rise in water.

Therefore,

$$\pi(6)^2(h) = 36\pi$$
$$36\pi h = 36\pi$$
$$h = 1\ cm$$

▶ Use the trapezoidal rule to approximate area.

Example: Use the trapezoidal rule to estimate the area under the curve shown.



Estimated area =

$$\frac{2}{2}[1 + 8 + 2(2 + 4 + 6)]$$

$$= 9 + 2(12)$$

$$33cm^2$$

FINALLY

Stay focused and confident. Work hard and consistently. Maths needs to be done every day from now until you finish Paper 2!

Good Luck!!

Attention to detail is key to Ordinary Level exam success

Building a strong foundation in Maths is important to any future career. Maths teacher at Yeats College ***Tomás Keane*** *provides guidance on how to achieve your best in the Ordinary Level exam*

Structure of the Exam

The Ordinary Level Maths course is split into five strands:

Strand 1 – Probability and Statistics
Strand 2 – Geometry and Trigonometry
Strand 3 – Number
Strand 4 – Algebra
Strand 5 – Functions and Calculus

Each strand examines a different area of maths. Strands 1 and 2 are found in Paper 2 and Strands 3, 4 and 5 are found in Paper 1.

To accommodate the lack of class time over the past year, the layout of both papers has changed. Section A will contain six questions, as usual. However, the marks per question in Section A will increase to 30 marks, and candidates will be required to answer any four of the questions. In Section B, there will be four questions at 50 marks each, and candidates will be required to answer any two of them.

The total mark allocation for each examination paper will be 220 marks, instead of the usual 300 marks. Each paper is still 2 hours and 30 minutes long, which means that candidates have an additional 40 minutes to work with so time management should not be an issue.

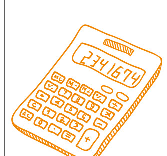
Both papers are split into two sections:

Section A – Concepts and Skills (120 marks): This section examines students' knowledge of various concepts and methods. These questions are more straightforward and formula heavy.

Section B – Context and Applications (100 marks): This section asks students to apply their mathematical skills to real-world situations. The marking for the questions can vary from year to year.

Tips for the Exam

- **Before the exam,** check that your calculator is working and that you have a compass, ruler, protractor, set squares, a pencil and different coloured pens. Also check the location of the formulas in the *Formulae and Tables Booklet (Maths Tables)*. More importantly, make a list of formulas and facts that are not in the tables. Make sure that you know these before sitting the exam.
- **Time management:** The maximum time you should spend on any question is half the number of marks for that question. For example, you should spend 25 minutes on one of the long questions in Section B.
- **Extra questions:** I would recommend answering as many questions as possible. This will help maximise the number of marks in both papers, and will ultimately give students their best chance of success in June.
- **Memory wipe:** At the start of the exam, I recommend that students do a memory wipe. There are several formulas and facts that students need to learn before going into the exam. Unfortunately, students go blank during the exam. To counteract this, start the exam by writing down the memorised material on the extra paper.
- **Highlight the key words** of the question and write down the formulas or key facts that are associated with that key word.
- **Start the question by writing the relevant formula.** Marks are awarded for any correct step in the method and for showing work of merit.
- **Blank spaces:** Never leave a blank space in Maths. It is essential for students to attempt the question in order to pick up some attempt marks. The marking scheme is designed so that students will be awarded marks for getting the steps of the method correct. Show your workings throughout.



Before the exam, check that your calculator is working and that you have a compass, ruler, protractor...

- **Never use Tipp-Ex or an eraser.** Simply put a line through your work and move on. There is plenty of extra paper available at the back of the exam paper.
- **Scanned exam:** Answer the questions in the space provided. There is a dark outline around the working space, so don't go outside these lines. The exam will be scanned through a system so a part of your work might not be visible. There is extra paper at the back of the exam.
- **Units of measurement:** It is very easy to forget the units of a question. Always check the units of your solution. This is particularly important for questions involving area and volume.
- **Language of Maths:** Think about the language of the question. One of the skills that I encourage students to improve upon is their competency in relation to the language of Maths. Always highlight the keywords of a question and write down the key point/formula for that keyword.
- **Logic and mathematical reasoning:** Always check your answers in relation to the original question. Your answer might not make logical sense.
- **Start the exam with the questions that you find easiest.** This will help build your confidence in the exam. Finish the exam with your least favourite topic.

Paper

Number Theory

1 Natural Numbers: This is the set of positive whole numbers: $\mathbb{N} = \{1,2,3,4,5,\dots,+\infty\}$.

2 Integers: This is the set of positive numbers, negative numbers and the number zero: $\mathbb{Z} = \{-\infty,\dots,-2,-1,0,1,2,\dots,+\infty\}$.

3 Rational Numbers: A rational number (i.e., a fraction) is a number of the form $\frac{a}{b}$, where a and b are both integers, with $b \neq 0$. In other words, a rational number is a ratio of two integers. We use $\mathbb{Q}$ to represent rational numbers. Examples: $\frac{1}{2}$, 0.7.

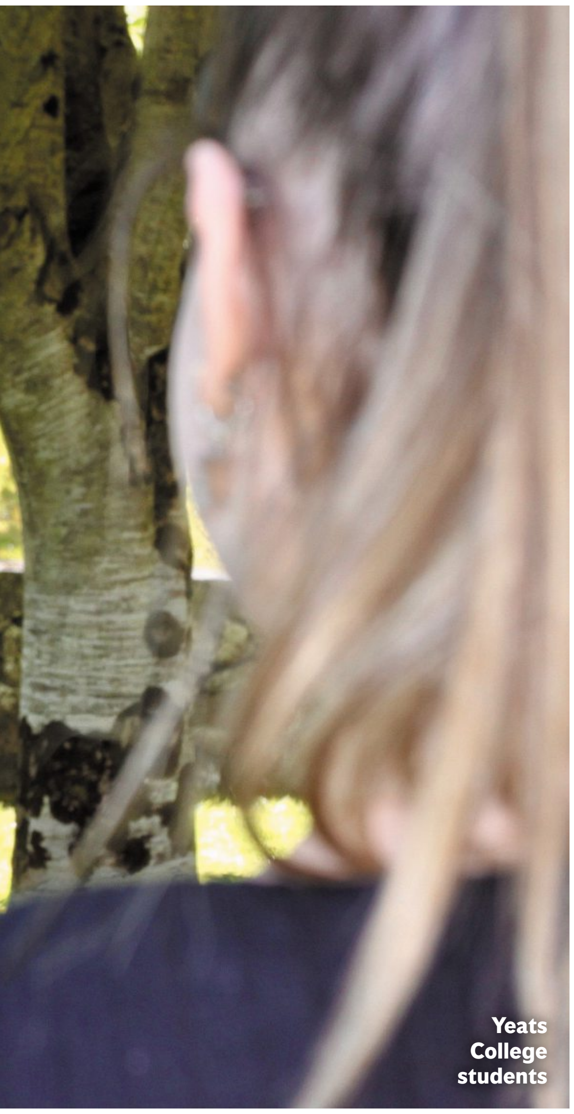
4 Irrational Numbers: These are all real numbers with the exception of rational numbers. We use $\mathbb{R} \setminus \mathbb{Q}$ to denote this. In simple terms, irrational numbers are numbers that go on forever, i.e., they cannot be written on the page. Examples: $\pi, \sqrt{2}$

5 Real Numbers: This is the set of all numbers including decimals and fractions. All numbers are described as real numbers, unless they are complex numbers. We use R to represent real numbers.

$$\mathbb{R} = \{-\infty \le x \le \infty\}$$

Other important concepts

6 Prime Numbers: These are numbers that have only 2 factors; the number itself and 1. Prime Numbers: {2,3,5,7,11,13,17,...}



Yeats College students

One

7 Lowest Common Multiple: The **LCM** means the lowest common multiple. *Example:* The LCM of 5 and 7 is 35 as it is the lowest number that both 5 and 7 divides into evenly.

8 Highest Common Factor: The **HCF** is highest common factor. *Example:* The HCF of 9 and 7 is 63 as 63 is the highest number that divides into both 9 and 7 evenly.

9 Absolute Value (Modulus) of a number: |Number| = positive value of that number.

10 Scientific Notation: $a \times 10^n$, where $n \in \mathbb{N}$ and $1 \leq a < 10$.

11 Percentage Error = $\frac{\text{Exact Value} - \text{Approximate Value}}{\text{Exact Value}} \times 100$

12 Significant Figures: To round off a number to a given number of significant figures, change the number into scientific notation (i.e. $a \times 10^n$) first. The number of digits in 'a' corresponds to the specified number of significant figures.

13 Distance, Speed and Time: Distance = Speed ×Time;

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} \text{ ; Time} = \frac{\text{Distance}}{\text{Speed}}$$

Complex Numbers

Complex numbers are used to overcome any issue involving the square roots of negative numbers. We use $i = \sqrt{-1}$, where i (known as iota) represents an imaginary number. From this, we can see that $i^2 = -1$.

What is a complex number?

A complex number z is a number that can be written in the form $z = a + bi$, where $a, b \in \mathbb{R}$ and $i = \sqrt{-1}$. The variable a is called the real part of the complex number and the variable b is called the imaginary part of the complex number. In simple terms, a complex number = real part + (imaginary part) × i.

Addition, Subtraction and Multiplication of Complex Numbers

These questions are student friendly. All you have to do is group the real parts together and then group the imaginary parts together. Simply add the real numbers with the real numbers and then add the imaginary numbers with the other imaginary numbers. When multiplying complex numbers, ensure that you multiply out i^2, i^3, i^4, ... using $i^2 = -1$.

Division of Complex Numbers and the Complex Conjugate

Method for Dividing Complex Numbers: Multiply above and below by the complex conjugate of the denominator. Our objective is to ensure that there is no imaginary number in the denominator.

What is a complex conjugate? The conjugate of a complex number is the same complex number except that you have to change the sign of the imaginary part. We use $\bar{z}$ to denote the conjugate.

If $z = a + bi$ is a complex number, then $\bar{z} = a + bi$ is called the ***complex conjugate of z***.

2016 Paper 1 – Question 2

Part (a) and part (d) examine the addition, subtraction, multiplication and division of complex numbers.

$z_1 = 1 + 3i$ and $z_2 = 2 - i$, where $i^2 = -1$, are two complex numbers.

(a) Let $z_3 = z_1 + 2z_2$. Find z_3 in the form $a + bi$, where $a, b \in \mathbb{Z}$.

Sub in the values for z_1 and z_2. Multiply out the brackets, then add the reals with the reals and then the imaginaries with the imaginaries.

$$z_3 = z_1 + 2z_2$$
$$= (1 + 3i) + 2(2 - i)$$
$$= 1 + 3i + 4 - 2i = 5 + i$$

(b) Find the complex number w, such that $w = \frac{z_1}{z_2}$. Give your answer in the form $a + bi$, where $a, b \in \mathbb{R}$.

$$w = \frac{z_1}{z_2} = \frac{1+3i}{2-i}$$

Multiply top and bottom by the conjugate of the denominator. Here, the conjugate is $2 + i$.

$$= \frac{(1+3i)(2+i)}{(2-i)(2+i)}$$

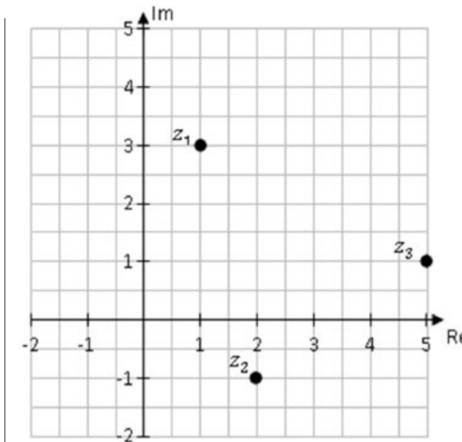
$$= \frac{2 + i + 6i + 3i^2}{4 + 2i - 2i - i^2} = \frac{-1 + 7i}{5} = -\frac{1}{5} + \frac{7}{5}i$$

Always separate the larger fraction into two so that your answer is written in the form a+bi.

The Argand Diagram

Just as real numbers can be represented by points on the real number line, you can represent a complex number $z = a + bi$ at the point (a, b) in a co-ordinate plane (the complex plane). This is known as the **Argand Diagram**. The horizontal axis is called the **real axis** and the vertical axis is called the **imaginary axis**.

The Argand diagram (*above right*) is from the 2016 exam question. In that question, students were asked to plot and label three complex numbers ($z_1 = 1 + 3i$, $z_2 = 2 - i$ and $z_3 = 5 + i$) on the Argand diagram. Complex Numbers are plotted in exactly the same way as points in coordinate geometry except that there are i's in the vertical axis. Finally, the origin is written as 0 + 0i.



The Modulus

Modulus of a Complex Number: The modulus (or absolute value) of the complex number $z = a + bi$ is defined as the distance between the origin (0, 0) and the point (a, b).

Formula: If $z = a + bi$, then

$$|z| = |a + bi| = \sqrt{a^2 + b^2}$$

In words: $\sqrt{(\text{real part})^2 + (\text{imaginary part})^2}$

Example: If $z = 5 - 12i$, then the modulus is

$$|z| = |5 - 12i| = \sqrt{(5)^2 + (-12)^2}$$
$$= \sqrt{25 + 144} = \sqrt{169} = 13 \text{ units.}$$

Finally, the formula above is not in the Maths tables. Therefore, students will need to learn it off by heart.

2018 Paper 1 – Question 2

Let $z_1 = -2 + 3i$ and $z_2 = -3 - 2i$, where $i^2 = -1$. $z_3 = z_1 - z_2$. Investigate if $|z_3| = |z_1| + |z_2|$.

$$|z_1| = \sqrt{(-2)^2 + (3)^2} = \sqrt{13} \text{ units.}$$
$$|z_2| = \sqrt{(-3)^2 + (-2)^2} = \sqrt{13} \text{ units.}$$
$$z_3 = z_1 - z_2 = -2 + 3i + 3 + 2i = 1 + 5i.$$
$$|z_3| = \sqrt{(1)^2 + (5)^2} = \sqrt{26} \text{ units.}$$
$$|\boldsymbol{z_3}| \neq |\boldsymbol{z_1}| + |\boldsymbol{z_2}| \text{ since}$$
$$\sqrt{26} \neq \sqrt{13} + \sqrt{13}.$$

Algebra

The best students in Maths are those that have a strong knowledge of algebra. Being proficient in algebra will help students succeed in this subject. Algebra is fundamental across all aspects of the subject and students will need to use the techniques of this section to help solve real-world problems in Section B.

Solving Linear Equations

What is an Equation? An equation is a mathematical statement with two sides. An equation consists of a left-hand side (LHS) and a right-hand side (RHS) with an equality sign (=) between the sides.

What does the answer represent? The answer is called the solution (or root) of the equation. The solution of an equation is the number that makes **both sides balance**. There is only one solution to a linear equation. Every linear equation has one root only.

2017 Paper 1 – Question 4(a)

Solve for x:

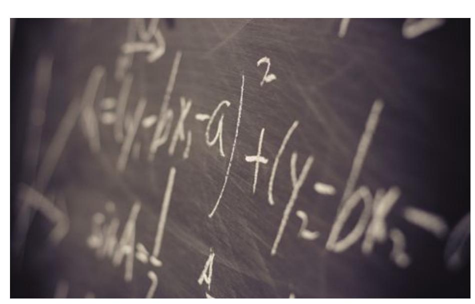
$$11x - 5(2x - 1) = 3(6 - x) + 3$$

Solution: Multiply out the brackets first.

$$11x - 10x + 5 = 18 - 3x + 3$$

Group the x's on the left and group the numbers on the right.

$$11x - 10x + 3x = 18 + 3 - 5$$
$$4x = 16$$
$$x = 4$$



Tip: Quickly verify your answer by substituting your answer into the original equation. Both sides of the equals sign will balance if the solution is correct.

$$11(4) - 5(2(4) - 1) = 3(6 - (4)) + 3$$
$$44 - 35 = 6 + 3$$
$$9 = 9$$

Therefore, $x = 4$ is the solution to the equation.

Solving Quadratic Equations

What is a quadratic equation? Any equation of the form $ax^2 + bx + c = 0$, $a \neq 0$, is called a quadratic equation. Solving a quadratic equation gives us the root(s) of the equation. The roots of an equation are where the quadratic curve crosses the x-axis. These x-values are the values which satisfy the equation.

In order to **solve a quadratic equation**, you must first factorise the expression and then let each factor equal to zero and hence solve. There are four methods of factorisation for quadratic equations:

1. Take out common terms:
E.g. $3x^2 + 24x = 0$
. $3x(x + 8) = 0 \Rightarrow x = 0$ and $x = -8$

2. Difference of two squares:
E.g. $x^2 - 49 = 0$
. $(x + 7)(x - 7) = 0 \Rightarrow x = -7$ and $x = 7$

3. Factorise a quadratic expression:
E.g. $x^2 - 5x + 4 = 0$
. $(x - 1)(x - 4) = 0 \Rightarrow x = 1$ and $x = -4$

4. Quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If you struggle with factorisation, I encourage you to use the quadratic formula. This formula can be found on page 20 of the Maths tables.

Simultaneous Equations – Type 1

What are simultaneous equations? When two equations are both satisfied by the same values of x and y, they are said to be simultaneous equations. The word simultaneous implies that the equations are happening at the same time so it makes sense that the answers work for both equations.

There are two types of simultaneous equations for Ordinary Level:
1. Two linear equations
2. One linear equation and one non-linear equation.

More Algebra, overleaf ▶▶▶

Algebra *(continued)*

2018 Paper 1 – Question 3(b)
Solve the simultaneous equations below to find the value of a and the value of b.

$$2a + 3b = 15$$
$$5a + b = -8$$

Rewrite the equations so that the variables match up. Thankfully, this is already done in the question. In this question, I will eliminate the b's by multiplying the top line by 2 and the bottom line by 3.

$$2a + 3b = 15 \quad (\times 1) \Rightarrow \quad 2a + \cancel{3b} = 15$$
$$5a + b = -8 \ (\times -3) \Rightarrow -15a - \cancel{3b} = 24$$
$$\Rightarrow -13a = 39 \Rightarrow a = -3$$

Now, substitute $a = -3$ into one of the original equations in order to find b:

$$a = -3 \Rightarrow 2(-3) + 3b = 15$$
$$-6 + 3b = 15 \Rightarrow 3b = 21 \Rightarrow b = 7$$

You could also check your answer (i.e. **verify** your answer) by subbing both answers into the original equations. Both equations should balance if the answers are correct.

Simultaneous Equations – Type 2

The second type of simultaneous equations has one equation that is linear and one equation that is non-linear (either quadratic or circular).

Method for solving these equations:

1. From the linear equation, express one variable in terms of the other.
2. Substitute this into the non-linear equation and solve.
3. Substitute separately the value(s) obtained in Step 2 into the linear equation in Step 1 to find the corresponding value(s) of the other variable.

2019 Paper 1 – Question 4(b)
Solve the simultaneous equations:

$$x - 5y = -13$$
$$x^2 + y^2 = 13$$

The best way to tackle this question is to break it down into three simple steps.

Solution:
Step 1 – Rewrite the linear equation:

$$x - 5y = -13$$
$$\Rightarrow x = 5y - 13$$

It is much easier, algebraically speaking, to rewrite the equation for x. Rewriting for y would involve fractions and it is not recommended.

Step 2 – Sub this equation into the circle equation:

$$x^2 + y^2 = 13$$
$$(5y - 13)^2 + y^2 = 13$$
$$(5y - 13)(5y - 13) + y^2 = 13$$
$$25y^2 - 65y - 65y + 169 + y^2 = 13$$
$$26y^2 - 130y + 156 = 0$$

Divide the entire line by 26.

$$y^2 - 5y + 6 = 0$$
$$(y - 2)(y - 3) = 0$$
$$\Rightarrow y = 2 \text{ and } y = 3$$

Step 3 – Substitute these values for y into the rewritten linear equation from Step 1:

$$y = 2 \Rightarrow x = 5(2) - 13 = -3 \quad \therefore (-3, 2)$$
$$y = 3 \Rightarrow x = 5(3) - 13 = 2 \quad \therefore (2, 3)$$

This concept could also appear on Paper 2 in relation to The Line and The Circle. The answer above represents the **points of intersection** of the line $x - 5y = -13$ and the circle $x^2 + y^2 = 13$.

Inequalities

What are inequalities? Algebraic expressions that are linked by one of the four inequality symbols are called inequalities. The four inequality symbols are $<$, $\leq$, $>$ and $\geq$.

How to solve an inequality: Solving inequalities share the same method as solving equations, except with one main difference.
Important rule: The inequality sign is **reversed** when both sides are multiplied or divided by the same negative number. Finally, inequalities can be represented on the number line.

2019 Paper 1 –Question 6(a)
Solve the following inequality for $x \in \mathbb{R}$ and show your solution on the number line below:

$$2(3 - x) < 8$$

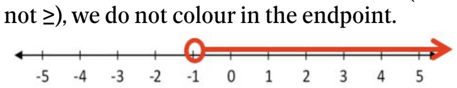
Solution:
Treat the inequality like an equation until near the end. Isolate the variable x by moving the constants to the other side of the inequality. Multiply out the brackets first.

$$6 - 2x < 8$$
$$-2x < 2$$

We have to divide both sides by –2.
The next line is the most important part of inequalities. If we divide (or multiply) by a negative number, we must **change the direction** of the inequality symbol. Hence,

$$x > -1$$

Since we have $x \in \mathbb{R}$, we must draw a bold line on the number line. Since we have > (and not ≥), we do not colour in the endpoint.



Indices

What are indices? An index (plural is indices) is another word for a power or exponent. Example: $4^3 = 4 \times 4 \times 4$.

What are exponential expressions? An exponential expression is an expression of the form a^p, where $p \in R$ and a is called the base of the expression.

What are exponential equations? An exponential equation is an equation where the unknown variable is a power.

2020 Paper 1 – Question 4(b)
Solve the equation

$$2^{(9x-1)} = 8^{2x}$$

In this question, you want to have the same base number on both sides of the equation. Here, change the 8 using indices ($8 = 2^3$).

$$2^{(9x-1)} = (2^3)^{2x}$$

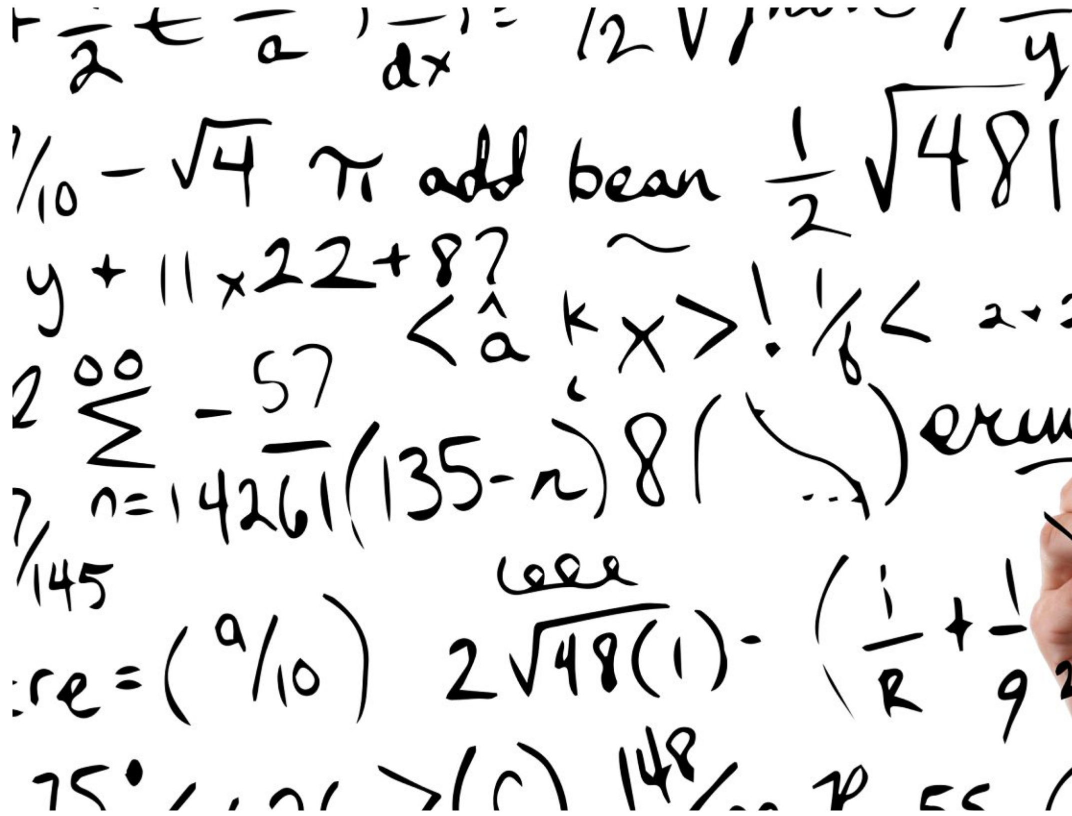
Multiply the power on the right-hand side since we have a power of a power.

$$2^{(9x-1)} = 2^{6x}$$

Now bring the powers down and solve the linear equation.

$$9x - 1 = 6x \Rightarrow 3x = 1 \Rightarrow x = 1/3$$

Optional tip: Verify your answer quickly by subbing your answer in the original equation. Both sides should balance.



Sequences and Series

The formulas for Sequences and Series are found on page 22 of the Maths tables.

Different Types of Sequences

Arithmetic sequences: An arithmetic sequence is a sequence in which the difference of the numbers is the same. Arithmetic sequences are sometimes known as linear sequences.

Example: 3, 7, 11, 15, 19, ...

The difference between terms in the above sequence is +4. Every number can be found by adding +4 to every term.

Geometric sequence: A geometric sequence is a sequence in which you go from term to term by multiplying or dividing by the same number.

Example: 5, 10, 20, 40, 80, 160, 320, ...

The difference between terms in the above sequence is ×2 .

Quadratic sequences: Sequences that have an nth term containing n^2 as the highest power are called quadratic sequences. If the second difference is constant, then the sequence is quadratic.

Example: 1, 4, 9, 16, 25, 36, ... In this sequence, the first difference is 3, 5, 7, 9, 11, ... and the second difference is 2, 2, 2, 2, ... If the second difference is constant (i.e. the same number each time), we can conclude that the sequence is quadratic. If the first difference were constant, then the sequence would have been linear (i.e. arithmetic).

Arithmetic Sequence – Formula

The nth term of an arithmetic sequence is given by

$$T_n = a + (n - 1)d$$

- The first term is denoted by a.
- The common difference between terms is denoted by d.
- Common difference = any term — previous term
- If you have a and d, you can find any term of the sequence.

This formula is all about the location of terms.

Example: Which term has a value of 126 in the arithmetic sequence 4, 6, 8, ...?
Theoretically, you could write out all of the terms until you get to the number 126. The faster and more efficient approach is the T_n formula. Here, the first term is 4 and the difference is 2. Hence, $a = 4$ and $d = 2$ respectively. Finally, we want to find the term that has a value of 126. Hence, $T_n = 126$.

Solution:

$$T_n = a + (n - 1)d$$
$$126 = 4 + (n - 1)(2)$$
$$126 = 4 + 2n - 2$$
$$124 = 2n \Rightarrow n = 62$$

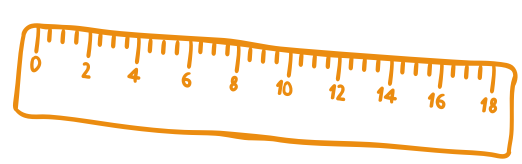
This means that the 62nd term has the value 126.

What is the difference between a sequence and a series? The only difference between a sequence and a series is that the comma signs are replaced with plus signs.

For example, 7, 14, 21, 28, 35, ..., is an arithmetic sequence whereas 7 + 14 + 21 + 28 + 35 + ..., is an arithmetic series.

We use the symbol S_n to denote the sum of the first n terms of a series. For example, S_{12} is the sum of the first 12 terms of the sequence.

A **series** is the sum of the individual terms of a sequence.



Financial Maths

Financial Maths for ordinary level is comprised of the following topics:

1. Percentage Profit & Percentage Loss:

$$\text{Percentage Profit} = \frac{\text{Profit}}{\text{Cost Price}} \times 100$$

$$\text{Percentage Loss} = \frac{\text{Loss}}{\text{Cost Price}} \times 100$$

2. Currency Exchange: Questions involving currency rates will involve multiplication and division.

2015 Paper 1 – Question 1 (a)(b)
Padraic works in America and travels between Ireland and America.

(a) In Ireland, he exchanged €2,000 for US dollars when the exchange rate was €1 = $1.29. Find how many US dollars he received.

In this question, simply multiply his money by the exchange rate.

€2,000 × 1.29 = $2,580

(b) Padraic returned to Ireland and exchanged $21,000 for euro. He received €15,000. Write the exchange rate for this transaction in the form €1 = $☐.☐☐

€15,000 = €21,000

$$€1 = \frac{21000}{15000} = \$1.40$$

Thus, the exchange rate is €1 = $1.40

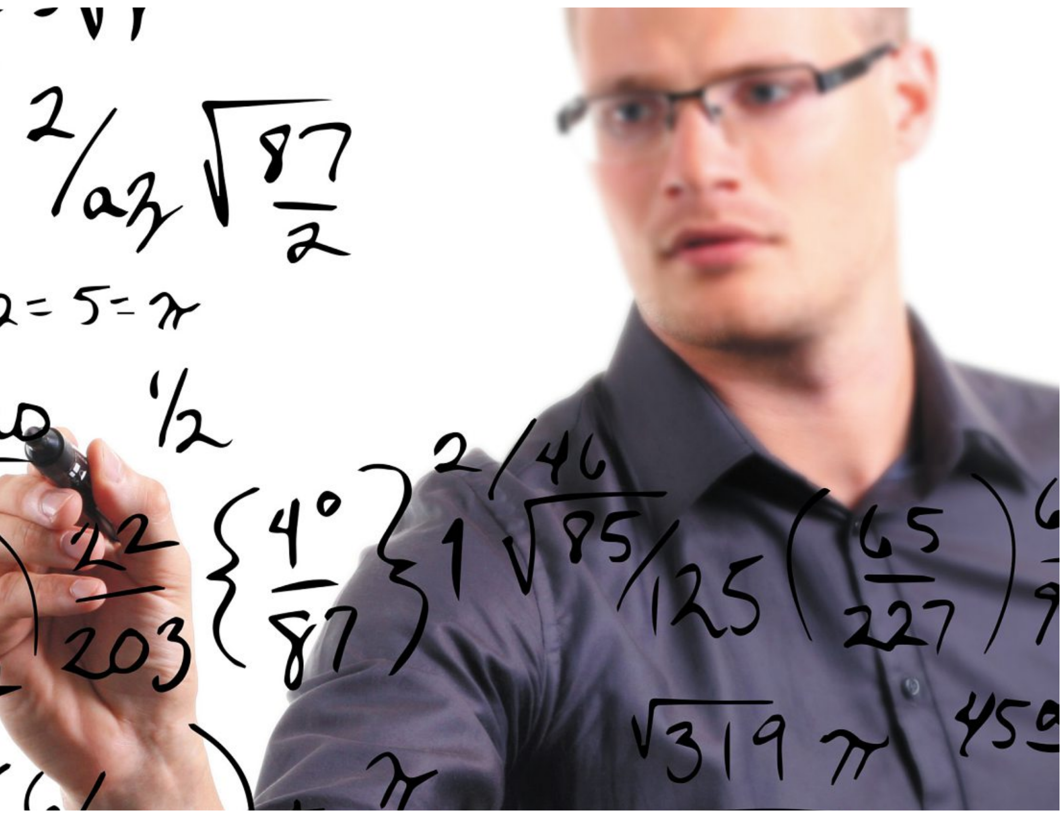
3. Income tax: Questions will involve VAT, PRSI, USC and tax credits.

4. Compound interest: The formula for compound interest is

$$F = P(1 + i)^t$$

5. Depreciation: The formula for depreciation is the same as compound interest, except you have to swap the plus sign with a minus sign.

$$F = P(1 - i)^t$$



Arithmetic Series – Formula

The sum to n terms of an arithmetic series is given by the formula

$$S_n = \frac{n}{2}[\,2a + (n-1)d\,]$$

- The first term (i.e. T_1) is denoted by a.
- The common difference between terms is denoted by d.
- n = the number of terms to be added.
- d = Common difference = any term — previous term.

2015 Paper 1 – Question 6

The exam question examines students' knowledge of the T_n and S_n formulas.

The general term of an arithmetic sequence is $T_n = 15 - 2n$, where $n \in \mathbb{N}$.

(a) (i) Write down the first three terms of the sequence.

The first three terms are also known as T_1, T_2 and T_3. Substitute $n = 1$, $n = 2$ and $n = 3$ into the T_n formula.

$T_1 = 15 - 2(1) = 15 - 2 = 13$

$T_2 = 15 - 2(2) = 15 - 4 = 11$

$T_3 = 15 - 2(3) = 15 - 6 = 9$

The first three terms are 13, 11 and 9.

(a) (ii) Find the first negative term of the sequence.

The sequence above is an arithmetic sequence since the difference between each term is the same. This sequence is decreasing by 2 with each term. Continue the pattern until you reach the first negative number.

13, 11, 9, 7, 5, 3, 1, – 1.

Therefore, the eighth term contains the first negative term of the sequence, i.e. $T_8 = -1$. You could also use inequalities to solve this problem. The problem can be written as $T_n < 0$.

(b) (i) Find $S_n = T_1 + T_2 + \ldots + T_n$, the sum of the first n terms of the series, in terms of n.

First of all, highlight 'in terms of n'. This tells the student that their answer will have n in it. This question wants students to find a general formula for the sum of the first n terms. Start these questions by filling in the values of a and d. The first term is $a = 13$ and the difference is $d = -2$. Sub these values into the S_n formula:

$$S_n = \frac{n}{2}[\,2a + (n-1)d\,]$$

$$= \frac{n}{2}[\,2(13) + (n-1)(-2)]$$

$$= \frac{n}{2}[\,26 - 2n + 2\,]$$

$$= \frac{n}{2}[\,28 - 2n\,] = 14n - n^2$$

(ii) Find the value of n for which the sum of the first n terms of the series is 0.

The question wants the student to use the formula $S_n = 0$.

$S_n = 14n - n^2$

$S_n = 0 \Rightarrow 14n - n^2 = 0$

$\Rightarrow n^2 - 14n = 0$

Factorise by taking out the common terms.

$\Rightarrow n(n - 14) = 0$

$n = 0$ and $n = 14$.

The sum of the first n terms cannot be zero. Therefore, the answer is 14.

Finally, what if there was a part where they wanted the student to find the sum of the first 100 terms? All you have to do is substitute n = 100 into the S_n formula.

2013 Paper 1 – Question 4

Michael has a credit card with a credit limit of €1,000. Interest is charged monthly at 1.5% of the amount owed. Michael gets a bill at the end of each month. At the start of January, Michael owes €800 on his credit card. If Michael makes no repayments and no more purchases, show that he will exceed his credit limit after 15 months.

The formula for compound interest is

$$F = P(1 + i)^t$$

where F = Final Amount, P = Principal, i = Interest and t = Number of Months. In this question, $P = 800$, $t = 15$ and $i = 0.015$ (this is 1.5% written as a decimal since the formula requires it).

$$F = 800(1 + 0.015)^{15}$$
$$= 800(1.015)^{15} = 1000.18$$

Notice how €1,000.18 > €1,000. This means that Michael will exceed his credit limit after 15 months.

Functions & Graphs

Key terms for functions

Input: An input is a value that is put into the function. This is the x value.

Output: An output is the value that we receive from the input once it is inserted into the function. This is the $f(x)$ value.

Domain: The domain is the set of input values.

Range: The range is the set of actual output values.

Codomain: The codomain is the set of all possible output values.

Couples/ordered pairs: Functions may be written as a set of couples or ordered pairs, i.e. (input, output).

How to write a function: There are three ways to write a function. The following are all equivalent:

$$f: x \to 2x + 10$$
$$f(x) = 2x + 10$$
$$y = 2x + 10$$

There are four types of functions for Ordinary Level; linear, quadratic, cubic and exponential.

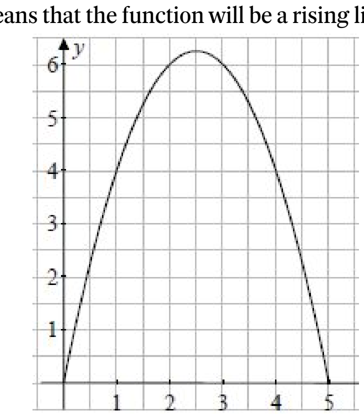
Type	Function
1. Linear	$f(x) = ax + b$
2. Quadratic	$f(x) = ax^2 + b + c$
3. Cubic	$f(x) = ax^3 + bx^2 + cx + d$
4. Exponential	$f(x) = ab^x$

2017 Paper 1 – Question 4(b)

The diagram shows the graph of the function $f(x) = 5x - x^2$ in the domain $0 \le x \le 5$, $x \in \mathbb{R}$.

(a) The function g is $g(x) = x + 3$, $x \in \mathbb{R}$. The points $A(1, k)$ and B are the points of intersection of f and g. Find the co-ordinates of A and B.

Solution: Before we begin, it is important to be able to roughly sketch the graph before starting the calculations. By looking at the function, we can determine whether the function is linear, quadratic, cubic or exponential and we can sketch the graph of each type of function. For example, the function $f(x)$ above is a negative quadratic function since the function contains a negative value before the x^2 term. The graph is outlined below. The function $g(x)$ is a positive linear function since the function contains a positive value before the linear term. This means that the function will be a rising line.

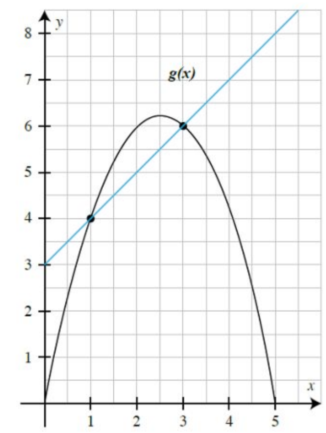


There are two methods for this question. The graphical approach is one where the student graphs both functions and hence finds the points of intersection from the graph itself. The second method, which is the algebraic method, uses simultaneous equations instead.

Graphical solution: Graph the linear function using the following table:

Domain x	Function $g(x) = x + 3$	Range y	Co-ordinates (x, y)
0	$g(0) = 0 + 3 = 3$	3	$(0, 3)$
5	$g(5) = 5 + 3 = 8$	8	$(5, 8)$

All functions can be graphed using the above function. The 1st column contains the domain (i.e., list of inputs), the 2nd column contains the function, the 3rd function contains the range (i.e., list of outputs) and the 4th column contains the co-ordinates. Plot the co-ordinates on the same diagram and then join the points with a line. From the graph we can see that both functions cross over at $A(1, 4)$ and $B(3,6)$.



Algebraic solution: Let $f(x) = g(x)$. Substitute both functions into the above equation and solve for x.

$f(x) = g(x)$

$5x - x^2 = x + 3$

$0 = x^2 - 4x + 3$

$0 = (x - 1)(x - 3)$

$x = 1$ and $x = 3$

$g(x) = x + 3$

$\underline{x = 1:}$

$g(1) = 1 + 3 = 4$

$\underline{x = 3:}$

$g(3) = 3 + 3 = 6$

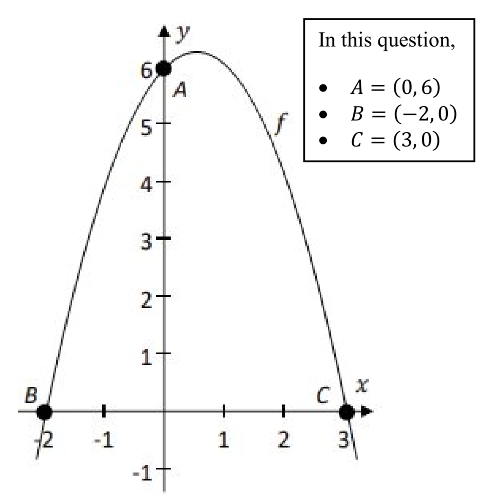
Thus, the points of intersection are $A(1, 4)$ and $B(3, 6)$.

2018 Paper 1 – Question 5

The diagram below shows the graph of a quadratic function f.

(a) Write down the co-ordinates of A = (,) = (,) C = (,)

(b) Show that the function can be written as $f(x) = -x^2 + x + 6$.



This question requires students to **form equations from their roots.** The roots of the equation above are $x = -2$ and $x = 3$. Rewrite by bringing the numbers to the left-hand side.

$$x + 2 = 0 \text{ and } x - 3 = 0$$

Find the factors and set equal to zero.

$$(x + 2)(x - 3) = 0$$

Multiply out the brackets.

$$x^2 - 3x + 2x - 6 = 0 \Rightarrow x^2 - x - 6 = 0$$

Move the entire equation to the right-hand side in order to makes the quadratic equation negative.

$$\Rightarrow -x^2 + x + 6 = 0$$

See 'Differentiation', overleaf ▶▶▶

Paper Two

Differentiation

Calculus is an important concept for students to grasp before going into Paper 1. Calculus is made up of two parts: differentiation and integration. However, integration does not appear at ordinary level. Differentiation, or differential calculus, is the branch of mathematics measuring **rates of change**.

If $y = f(x)$, then $\frac{dy}{dx}$ or $f'(x)$ gives the rate of change of y with respect to x. We use $\frac{dy}{dx}$ (or $f'(x)$) to denote the first derivative of y with respect to x.

General Rule for Differentiation

There is a simple method to follow for differentiation. When given a function comprised of a few terms, it is important to differentiate term by term and not all at once.

Type 1 – Terms involving powers
Say we are trying to differentiate $3x^2$. What we do is take the 2 down and multiply 3 by it. After that, we reduce the power by 1. The answer is $6x^1$ or $6x$.

Type 2 – Linear Terms
Let's try differentiating $-8x$. These questions are a lot simpler. All you have to do is forget about the x. The answer is -8.

Type 3 – Constant Terms
Finally, let's differentiate 30. This is the easiest question. The derivative of a constant is zero. The answer is 0.

Example: Differentiate the function $y = 4x^3 + 5x^2 - 3x + 12$.

Solution:

$$y = 4x^3 + 5x^2 - 3x + 12$$

$$\Rightarrow \frac{dy}{dx} = 12x^2 + 10x - 3 + 0$$

Evaluating Derivatives

Example: If $y = x^2 - 3x$, find the slope of the tangent to the curve at the point where $x = 3$.

Solution: First of all, the phrase 'find the slope of the tangent to the curve at the point' is the same as finding $\frac{dy}{dx}$. Be mindful of the language of differentiation.

$$\frac{dy}{dx} = 2x - 3$$

Now, substitute $x = 3$ into the derivative above.

$$\Rightarrow \frac{dy}{dx} = 2(3) - 3 = 6 - 3 = 3$$

This means that the slope of the tangent is 3.

Second Derivatives

The derivative of $\frac{dy}{dx}$, denoted by $\frac{d^2y}{dx^2}$, is called the second derivative of y with respect to x. It is pronounced 'dee two y, dee x squared'. The method to work out $\frac{d^2y}{dx^2}$ is the same as $\frac{dy}{dx}$. You simply have to repeat the method for the derivative, i.e. you **differentiate twice**.

2019 Paper 1 – Question 3
The function f is defined as $f : x \to -x^3 + 4x^2 + x - 2$, where $x \in \mathbb{R}$.
(b) Find the value of x for which $f''(x) = 0$, where $f''(x)$ is the second derivative of $f(x)$.
Differentiate $f(x)$ first and then differentiate that answer again.

1st Derivative: $f'(x) = -3x^2 + 8x + 1$
2nd Derivative: $f''(x) = -6x + 8$

$$f''(x) = 0 \Rightarrow -6x + 8 = 0$$
$$\Rightarrow -6x = -8 \Rightarrow x = \frac{4}{3}$$

Curve Sketching

- If the curve is increasing, then $\frac{dy}{dx} > 0$.
- If the curve is decreasing, then $\frac{dy}{dx} < 0$.
- If the curve is neither increasing nor decreasing, then $\frac{dy}{dx} = 0$. This means that the slope of the tangent at that point is zero.

2014 Paper 1 – Question 5
The function f is defined as $f : x \to x^3 + 3x^2 - 9x + 5$, where $x \in \mathbb{R}$.
(a) (i) Find the co-ordinates of the point where the graph of f cuts the y-axis.
All functions cross the y-axis when $x = 0$. Substitute this into the function above.

$$f(x) = x^3 + 3x^2 - 9x + 5$$
$$f(0) = (0)^3 + 3(0)^2 - 9(0) + 5 = 5$$

Thus, the y-intercept is $(0, 5)$.

(ii) Verify that the graph of f cuts the x-axis at $x = -5$.
A graph cuts the x-axis when $y=0$. In this question, substitute $x = -5$ into the function. This will result in the answer being zero.

$$f(5) = (-5)^3 + 3(-5)^2 - 9(-5) + 5$$
$$= -125 + 75 + 45 + 5 = 0$$
$$f(-5) = 0 \Rightarrow (-5, 0) \text{ is the } x\text{-intercept.}$$

(b) Find the co-ordinates of the local maximum turning point and the local minimum turning point of f.
The formula to use here is $f'(x) = 0$ (this is the same as $\frac{dy}{dx} = 0$). This formula is not in the Maths tables, so it is a formula that students must **learn**.

$$f'(x) = 0$$
$$f'(x) = 3x^2 + 6x - 9$$
$$f'(x) = 0 \Rightarrow 3x^2 + 6x - 9 = 0$$

Factorising the above quadratic equation yields:

$$\Rightarrow (3x + 9)(x - 1) = 0$$
$$\Rightarrow 3x + 9 = 0 \qquad x - 1 = 0$$
$$\Rightarrow 3x = -9 \qquad x = 1$$
$$\Rightarrow x = -3 \quad \text{and} \quad x = 1$$

Substitute these values of x into $f(x)$ in order to get the y-value of the co-ordinates.

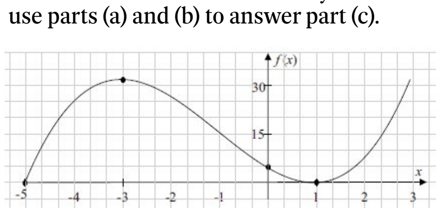
$x = 1$: $f(1) = (1)^3 + 3(1)^2 - 9(1) + 5$
$= 1 + 3 - 9 + 5$
$= 0 \quad \therefore (1, 0)$

$x = -3$:

$f(-3) = (-3)^3 + 3(-3)^2 - 9(-3) + 5$
$= -27 + 27 + 27 + 5$
$= 32 \quad \therefore (-3, 32)$

The turning point with the larger y-value is the maximum turning point. Since $32 > 0$, the point $(-3, 32)$ is the maximum turning point. Subsequently, $(1,0)$ is the minimum turning point.

(c) Hence, sketch the graph of the function f on the axes below.
The word 'hence' indicates that you must use parts (a) and (b) to answer part (c).



Rates of Change (i.e., Motion Questions)

Rates of change are questions involving real-world concepts such as distance (or height), speed, velocity, acceleration and deceleration. The best way to start questions involving rates of change is to find all three general formulas for distance, speed and acceleration. The distance (sometimes height) formula is typically given to you in the question. There are a few things that you need to learn for this section:

- Speed (or Velocity): $v = \frac{ds}{dt}$
- Acceleration: $a = \frac{d^2s}{dt^2}$ or $\frac{dv}{dt}$
- 'Comes to rest' means that speed = 0, i.e., $\frac{ds}{dt} = 0$.
- Maximum height means that $\frac{dh}{dt} = 0$.

The Line

The following six formulas are in the Maths tables:

1. **Distance** = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
2. **Midpoint** = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
3. **Slope:** $m = \frac{y_2 - y_1}{x_2 - x_1}$
4. **Equation of a line:** $y - y_1 = m(x - x_1)$
5. **Equation of a line #2:** $y = mx + c$
6. **Area of a triangle at the origin** = $= \frac{1}{2}|x_1y_2 - x_2y_1|$.

Key Points about The Line

The following are formulas and facts that must be learnt before going into the exam:

1. **X-intercept:** A line crosses the x-axis when $y = 0$. This is called the x-intercept.
2. **Y-intercept:** A line crosses the y-axis when $x = 0$. This is called the y-intercept.
3. **Slope:** The slope can be found using $m = \frac{y_2 - y_1}{x_2 - x_1}$ or $\frac{Rise}{Run}$ or $-\frac{a}{b}$.
4. **Parallel Lines:** Parallel lines have the same slope.
5. **Perpendicular Lines:** Perpendicular lines have inverted slopes (i.e. they're perpendicular).
6. **Perpendicular Slopes:** Perpendicular slopes multiply to give minus one ($m_1 \times m_2 = -1$).
7. **Graphing Lines:** When graphing lines, use $x = 0$ to find the y-intercept and $y = 0$ to find the x-intercept. These are the easiest points to use.
8. **Point of Intersection:** There are two methods:
   - *Algebraic Method:* Use simultaneous equations to find the point of intersection of two lines.
   - *Graphical Method:* Graph both lines using $x = 0$ and $y = 0$. Use the graph to find the point of intersection.

2019 Paper 2 – Question 2
The diagram (*opposite page*) shows the line PQ and the line QR. The co-ordinates of the points are $P(4, 2)$, $Q(8, 5)$ and $R(2, 11)$.
(a) Find the slope of PQ

The Circle

Equation of a Circle

The most important section of co-ordinate geometry of the circle is the equation of a circle. There are two equations that you need to be aware of:

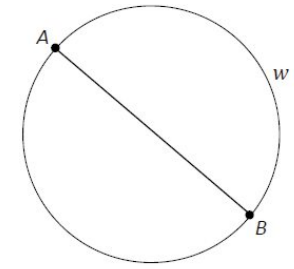
1. **Equation of a circle with centre (0, 0):** The centre of this circle is at the origin. The formula is $x^2 + y^2 = r^2$.
2. **Equation of a circle with centre (h, k):** The centre of this circle is not at the origin. The formula is
$$(x - h)^2 + (y - k)^2 = r^2$$

Key Points about The Circle

3. **Radius:** The radius can be found by using the distance formula
$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$
4. **Diameter:** Diameter = 2 × radius.
5. **Centre:** When given the two endpoints of the diameter, use the midpoint formula $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ to find the centre of the circle.
6. **X-intercept(s):** A circle crosses the x-axis when $y = 0$.
7. **Y-intercept(s):** A circle crosses the y-axis when $x = 0$.
8. **Points of Intersection:** The intersection of a circle and a line can be found algebraically using simultaneous equations (type 2).
9. **Points inside, on or outside a circle:** Sub the point into the given equation of a circle. If
   - LHS (Left-Hand Side) < RHS (Right-Hand Side), then the point is inside the circle.
   - LHS = RHS, then the point is on the circle.
   - LHS > RHS, then the point is outside the circle.
10. **Equation of a tangent to a circle:**
   - Find the slope from the centre to the point.
   - Find its perpendicular slope.
   - Use $y - y_1 = m(x - x_1)$ to find the equation of the tangent.

2018 Paper 2 – Question 4
This question combines the concepts of The Line and The Circle.

The points $A(1, 8)$ and $B(9, 0)$ are the end-points of a diameter of the circle w, as shown in the diagram.
(a) Find the co-ordinates of the centre of w.



Use the midpoint formula to find the centre.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{1+9}{2}, \frac{8+0}{2}\right) = \left(\frac{10}{2}, \frac{8}{2}\right) = (5, 4).$$

(b) Find the length of the radius of w. Give your answer in the form $p\sqrt{q}$ where $p, q \in \mathbb{N}$.

$$|BC| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(9 - 5)^2 + (0 - 4)^2}$$
$$= \sqrt{(4)^2 + (-4)^2}$$
$$= \sqrt{16 + 16} = \sqrt{32} = 4\sqrt{2}\text{units.}$$

(c) Hence write down the equation of the circle w.
Substitute the answers from part (a) and (b) into the equation of a line formula.

$$(x - h)^2 + (y - k)^2 = r^2$$
$$(x - 5)^2 + (y - 4)^2 = (\sqrt{32})^2$$
$$(x - 5)^2 + (y - 4)^2 = 32$$

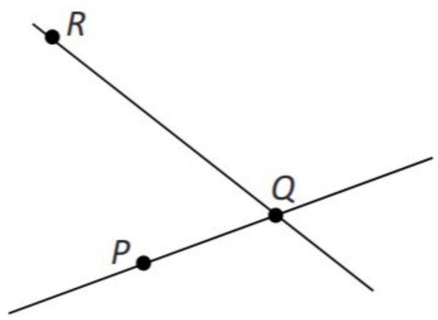
OR

$$x^2 + y^2 - 10x - 8y + 9 = 0$$

Use the slope formula from the maths tables.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-2}{8-4} = \frac{3}{4}$$

(b) Find the equation of the line *PQ*. Give your answer in the form *ax* + *by* + *c* = 0, where *a*, *b*, *c* ∈ ℤ.
Use the equation of a line formula from the Maths tables. Since ℤ (i.e. Integers) is mentioned, we have to write our equation using whole numbers. Fractions are not allowed.



$$y - y_1 = m(x - x_1)$$
$$y - 2 = \frac{3}{4}(x - 4)$$
$$3x - 4y - 4 = 0$$

(c) Write down the slope of any line perpendicular to *PQ*.
Flip the fraction and change the sign of the slope m_{PQ}. Ans = $-\frac{4}{3}$

(d) Find the area of the triangle *PQR*.
Use the formula $Area = \frac{1}{2}|x_1y_2 - x_2y_1|$ since we have three corner points. However, we must translate one of the points to the origin (0, 0). The remaining two points will also move with the translation.

(4 ,2) → (0, 0) (*x* ↓ 4 *spots*, *y* ↓ 2 *spots*)
(8, 5) → (4, 3)
(2, 11) → (–2, 9)

$$\text{Area} = \frac{1}{2}|(4)(9) - (-2)(3)| = \frac{1}{2}|36 + 6|$$

= 21 square units

The second answer given above can be found by multiplying out the brackets.

(d) Find the equation of the line that is a tangent to the circle *w* at *A*. Give your answer in the form *ax* + *by* + *c* = 0, where *a*, *b*, and *c* ∈ ℝ.
Recall that tangents are perpendicular to the radius that joins the centre of a circle to the point of tangency. This fact is used to find the slope of the tangent. Change the sign of the slope and flip the fraction upside down.
Use the slope formula to find the slope of the points *A* (1, 8) and *B*(9, 0).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-8}{9-1} = -\frac{8}{8} = -1$$

Thus, the perpendicular slope is *m* = +1. Sub this slope and the point *P*(1, 8) into the equation of a line formula:

$$y - y_1 = m(x - x_1)$$
$$y - 8 = 1(x - 1)$$
$$y - 8 = x - 1$$
$$0 = x - y + 7$$

Marks would be awarded for $y = x + 7$.

2014 Paper 2 – Question 3
This question combines the concepts of The Line and The Circle.
(a) (i) The circle *c* has equation $(x+2)^2 + (y-3)^2 = 100$. Write down the co-ordinates of *A*, the centre of *c*. Write down *r*, the length of the radius of *c*.

$$(x+2)^2 + (y-3)^2 = 100.$$

Centre is (-2, 3) $r = \sqrt{100} = 10$

For the centre, switch the sign of the numbers in the brackets. For the radius, find the square root of the value on the right-hand side. Leave this as a square root when possible. Do not change the number to decimal form.

(ii) Show that the point *P*(–8, 11) is on the circle *c*
Substitute the point (–8, 11) into the equation of a circle:

$$(-8+2)^2 + (11-3)^2 = 100$$
$$36 + 64 = 100$$
$$100 = 100$$

i.e. the left-hand side is **equal** to the right-hand side. Therefore, the point (–8, 11) is on the circle $(x+2)^2 + (y-3)^2 = 100$. It is very important to include this concluding statement as marks can be easily lost.

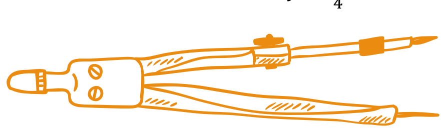
(b) (i) Find the slope of the radius [AP]
Use the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ with point (–2, 3) and (–8, 11)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11-3}{-8+2} = \frac{8}{-6} = -\frac{4}{3}$$

(ii) Hence, find the equation of *t*, the tangent to *c* at *P*
Recall that tangents are perpendicular to the radius that joins the centre of a circle to the point of tangency. This fact is used to find the slope of the tangent. Change the sign of the slope and flip the fraction upside down. Thus, the perpendicular slope is *m* = ¾. Sub this slope and the point *P*(–8, 11) into the equation of a line formula:

$$y - 11 = \frac{3}{4}(x + 8)$$
$$y - 11 = \frac{3}{4}x + 6$$
$$4y - 44 = 3x + 24$$
$$0 = 3x - 4y + 68$$

Marks would be awarded for $y = \frac{3}{4}x + 17$.

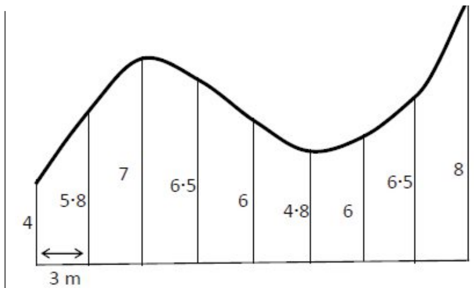


Area & Volume

Area and Volume is a concept that has the potential of appearing on both Paper 1 and Paper 2. Fortunately, most formulas are in the Maths tables. In simple terms, area is all about 2D shapes while volume is all about 3D shapes. Students often forget about the units in questions like these.
There are five main topics in this section:

1. **Area and Perimeter of 2D Shapes:** The Area and Perimeter formulas for 2D shapes are in the Maths tables.
2. **Surface Area and Volume of 3D Shapes:** The Surface Area and Volume formulas for 3D shapes are in the Maths tables. In this section, questions will involve rectangular boxes, prisms, cylinder, cones, spheres and hemispheres.
3. **Nets of Shapes:** When a 3D shape is opened out, the resulting flat shape is called the net of the shape.
4. **The Trapezoidal Rule:** The trapezoidal rule is used to estimate the area of an irregular shape.
5. **Other Questions** include Recasting (i.e., equal volumes), Immersion and Rate of Flow.

2017 Paper 1 – Question 6
A field is divided into eight sections as shown in the diagram (*above right*). The width of each section is 3 metres. The height, in metres, of each section is given in the diagram. Use the Trapezoidal Rule to estimate the area of the field.



How to use the Trapezoidal Rule

1. Divide the figure into a number of strips of equal width (the number of strips can be even or odd. Typically, the question does this for you).
2. Number and measure each height, h.
3. Use the following formula:

$$A \approx \frac{h}{2}[y_1 + y_n + 2(y_2 + y_3 + \cdots + y_{n-1})]$$

The above formula is difficult to understand. I recommend using words to help simplify things.

$$\text{Area} \approx \frac{\text{Step}}{2}[\text{1st Height} + \text{Last Height} + 2 \times (\text{Remaining Heights})]$$

$$= \frac{3}{2}[4 + 8 + 2(5.8 + 7 + 6.5 + 6 + 4.8 + 6 + 6.5)]$$

$$= 145.8\text{m}^2$$

Geometry, Construction & Enlargements

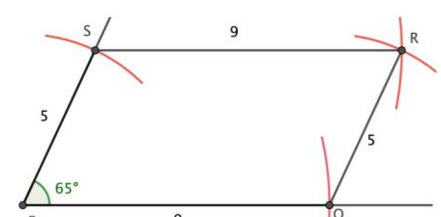
Geometry is the study of shapes. Key terms such as theorem, proof, axiom, corollary, converse and implies must be understood. I recommend learning examples for each key term.

Constructions

For this section, students will need a compass, a ruler, a protractor and set-squares. Students may be asked to perform one of the following constructions in Paper 2:

16. Circumcentre and circumcircle of a given triangle, using only straight-edge and compass.
17. Incentre and incircle of a given triangle, using only straight-edge and compass.
18. Angle of 60°, without using a protractor or set square.
18. Tangent to a given circle at a given point on it.
20. Parallelogram, given the length of the sides and the measure of the angles.
21. Centroid of a triangle.

The numbers above have been taken from the syllabus. See your textbook for further explanation. Constructions involving SSS, SAS, ASA and RHS can also appear. Students will not be asked to prove any of the theorems. Instead, students will be required to use the results of the theorems as explanation for their answers. In 2019, students were asked to construct a parallelogram.



Enlargements

1. **Drawing Enlargements:** To construct the image of a given figure under an enlargement, we need:
   - The **centre** of enlargement
   - The **scale factor** of the enlargement.
2. **Scale Factor:** The scale factor is the difference in size between two images (i.e., the ratio between the two images). When a shape is enlarged all the lengths are multiplied by the scale factor and all angles remain the same.
3. **Formula:** The scale factor, *k*, is given below:

$$k = \frac{\text{length of image side}}{\text{length of corresponding object side}}$$

4. **Area and Enlargements:** When a figure is enlarged by a scale factor *k*, the area of the image figure is increased by a scale factor k^2.

$$Area\ of\ Image = k^2 \times (Area\ Object)$$

$$\Rightarrow k^2 = \frac{\text{Area of Image}}{\text{Area of Object}}$$

Statistics

Statistics is the field of mathematics dealing with the collection, analysis, and presentation of data. Statistics is a theory-heavy concept. Students should familiarise themselves with the language and concepts surrounding statistics by using the textbook.

List of charts that students should know

Bar Charts, Scatter Plots, Line Plots, Pie Charts, Histograms, Positive Skew, Negative Skew, Normal Distribution, Back-to-back Stem and Leaf Diagrams, Correlation, Positive Correlation, No Correlation, Negative Correlation, Perfect Correlation.

Formulas and Facts for Statistics

1. **Mode:** This is the most common number in the list.
2. **Median:** When numbers are placed in ascending/descending order the median is the middle number.
3. **Mean:** This is the average of the list. To find the average, students must add up all the numbers and then divide by the number of numbers. The symbol for mean is $\boldsymbol{\mu}$.
4. **Range:** The highest value in a set of data minus the lowest value. This shows the spread of the data.
5. **Lower Quartile:** When data is arranged in descending/ascending order this is the value one quarter of the way into the data. The lower quartile is written as Q_1.
6. **Upper Quartile:** When data is arranged in descending/ascending order this is the value three quarters of the way into the data. The upper quartile is written as Q_3.
7. **Interquartile Range:** This is upper quartile minus the lower quartile. It can be found by calculating $Q_3 - Q_1$.
8. **Standard Deviation:** This is a measure of the spread of the values about the mean. The symbol is σ. The formulae are on page 33 in the Maths tables. In 2019, students were asked to find the standard deviation through their calculator for the first time.

Formulas

- Mean from a list of n numbers:

$$\mu = \frac{\sum x}{n}$$

(This means that you add up all of the numbers and then divide by the number of numbers.)

- Mean from frequency table:

$$\mu = \frac{\sum fx}{\sum f}$$

- Standard deviation from a list of n numbers:

$$\sigma = \sqrt{\frac{\Sigma(x-\mu)^2}{n}}$$

- Standard deviation from frequency table:

$$\sigma = \sqrt{\frac{\Sigma f(x-\mu)^2}{\Sigma f}}$$

Calculator: The mean and standard deviation can be found using STAT mode on the calculator. This is the preferred method for finding the standard deviation, and it was the required method in 2019.

Empirical Rule: For many large populations, the empirical rule provides an estimate of the approximate percentage of observations that are contained within one, two or three standard deviations of the mean.

• $\mu \pm 1\sigma$ covers **68%** of the sample
• $\mu \pm 2\sigma$ covers **95%** of the sample
• $\mu \pm 3\sigma$ covers almost all the sample **(99.7%)**.

What does Inferential Statistics mean?
Inferential Statistics is the branch of statistics that uses probability and statistics to draw conclusions from data that are affected by random variation. In this section, students should be able to:

1. Estimate the value of a **population proportion**.
2. Calculate the **margin of error** for a sample.
3. Construct a **Confidence Interval**.
4. Apply the **Empirical Rule** to Confidence Intervals.
5. Test a **hypothesis** about a population proportion.

2017 Paper 2 – Question 9(c)

In the days following the UK referendum, a survey was conducted in Ireland on attitudes towards a British exit from the European Union.

(i) 1,200 people were surveyed. Find the margin of error of this survey. Write your answer as a percentage. Give you answer correct to the nearest percent.

The formula for the margin of error is:

$$\text{Margin of Error} = \frac{1}{\sqrt{n}}$$

where n is the size of the sample.

$$\frac{1}{\sqrt{1200}} = 0.02887 = 2.887\% = 3\%$$

(ii) In the Irish survey 578 of the 1,200 surveyed agreed that a UK exit would have a negative effect on the Irish economy. Use your answer to part c(i) above to create a 95% confidence interval for the proportion of the Irish population who agreed that a UK exit would have a negative effect on the Irish economy.

A **Confidence Interval** for the true proportion at the 95% confidence interval is given by

$$\hat{\mathbf{p}} - \frac{1}{\sqrt{n}} \le \mathbf{p} \le \hat{\mathbf{p}} + \frac{1}{\sqrt{n}}$$

where n is the sample size, **p** is the population proportion and $\hat{\mathbf{p}}$ is the sample proportion. Simpler formula:

$$\textit{Sample Proportion} \pm \textit{Margin of Error}$$

We can say that the 95% confidence interval lies between inside this interval.

$$\frac{578}{1200} \times 100 = 48.1667 = 48\%$$

$$48 - 3 < p < 48 + 3$$

$$45 < p < 51$$

(iii) After the survey, a political party claimed that 53% of the Irish population felt that the decision of the UK to leave the EU would have a negative effect on the Irish economy. Use your answer to part c(ii) above to conduct a hypothesis test, at the 5% level of significance, to test the party's claim. Give your conclusion on the context of the question.

Null Hypothesis: $H_0 = p = 53\%$

Alternative Hypothesis: $H_1 = p \neq 53\%$

53% is not within the confidence interval, as found in **part (c)(ii)**.

$45.17 < p < 51.17$

Reject the null hypothesis H_0 and accept the alternative hypothesis H_1. To conclude our answer, we can now state that the political party's claim is false.

Probability

Probability is the branch of mathematics that uses numbers to describe how likely something is to happen. Probability is the study of the chance or likelihood of an event happening.

Key Points about Probability

1. Probability is measured on a scale from zero to one, i.e. $0 \le P(E) \le 1$. The probability of an impossibility is zero and the probability of a certainty is one. Probabilities always add up to 1.
2. **The Probability Formula:**

$$P(E) = \frac{\textit{number of successful outcomes}}{\textit{total number of possible outcomes}}$$

3. You can also calculate the **probability of an event not happening** using the formula below:
   $P(\textit{not } E) = 1 - P(E)$.
4. **Sample Space:** The set of all possible outcomes is called a sample space. *Example:* The sample space when throwing a die is {1, 2, 3, 4, 5, 6}.
5. **Experimental Probability:**

$$\text{Relative frequency} = \frac{\text{number of successful trials}}{\text{total number of trials}}$$

$$\text{Expected frequency} = (\text{prob}) \times (\text{no. of trials})$$

6. **Mutually Exclusive Events:** Events are mutually exclusive if they cannot occur at the same time.
7. **Independent Events:** Two events are independent if the outcome of one event does not affect the outcome of another.
8. **Dependent Events:** Events are dependent on each other if the outcome of one affects the outcome of another.
9. **The Addition Rule (The OR Rule):** When two events, A and B, are mutually exclusive then
   $P(A \text{ or } B) = P(A) + P(B)$
   When two events, A and B, are not mutually exclusive,
   $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
10. **The Multiplication Rule (The AND Rule):**
    $P(A \text{ and } B) = P(A) \times P(B)$.
11. **Bernoulli Trials:** Bernoulli Trials have two outcomes; success and failure.
12. **Probability and Venn Diagrams:** There is nothing particularly new about this section. The main difference is that the information for the question is presented using Venn Diagrams. Make sure that you are familiar with the terminology involved with Sets/Venn Diagrams.
13. **Probability and Tree Diagrams:** Tree diagrams can also be used to display possible outcomes of events. They can be used to show both independent and dependent events. The branches of the tree diagram represent the probabilities. Each pair of branches needs to add up to one, since the sum of probabilities add up to one.
14. **Expected Value:** The expected value is also known as the average outcome of an experiment. The formula to work out the expected value is

$$E(x) = \sum xP(x)$$

    This means we must add up the values of each outcome multiplied by the probability of getting that outcome.
15. **Fundamental Principle of Counting:** Suppose one operation has m possible outcomes and a different operation has n possible outcomes. The number of possible outcomes when performing the first operation followed by the second operation is $m \times n$.
16. **Permutations/Arrangements:** When you are asked about the number of permutations, you are being asked about the number of possible arrangements of events. Three objects can be arranged in 3! ways (pronounced factorial), i.e.
    $3! = 3 \times 2 \times 1 = 6$ ways

2019 Paper 2 – Question 6

In a population, the probability that a person has blue eyes is 0·7.

(a) One person is chosen at random from the population. What is the probability that this person does not have blue eyes?

$1 - 0.7 = 0.3$

(b) Two people are chosen at random. What is the probability that both have blue eyes?

The word 'and' is implied in the question. In this question, the first person **and** the second person are chosen. The word 'and' means multiply in Probability.

$0.7 \times 0.7 = 0.49$

(c) Three people are chosen at random. What is the probability that exactly two of them have blue eyes?

$P(\textit{exactly two}) = P(BBN)$ or $P(BNB)$ or $P(NBB)$
$= (0.7 \times 0.7 \times 0.3) + (0.7 \times 0.3 \times 0.7) + (0.3 \times 0.7 \times 0.7) = 0.441.$

(d) Four people are chosen at random, one after another. What is the probability that the fourth person of the four chosen is the only one to have blue eyes?

This question wants the student to find:
$P(NNNB) = 0.3 \times 0.3 \times 0.3 \times 0.7 = 0.0189.$

2018 Paper 2 – Question 3

(i) Find the number of different arrangements that can be made using all the letters of the word RAINBOW. Each letter is used only once.

Last year, the concept of arrangements (or permutations) made an appearance in the Probability question.

There are seven letters in the word RAINBOW. There are
$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 =$
5,040 ways of arranging seven letters.

(ii) Find the number of different three-letter arrangements that can be made using the letters of the word RAINBOW. Each letter is used at most once.

In this question, there are only three slots for which the letters could go. There are seven possibilities for the first slot. After this, there are six possibilities for the second slot. Finally, there are five possibilities. We multiply these three numbers by one other.

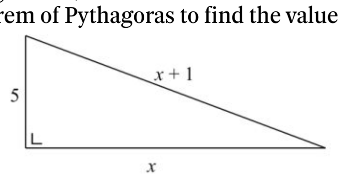
$7 \times 6 \times 5 = 210$ ways.

Trigonometry

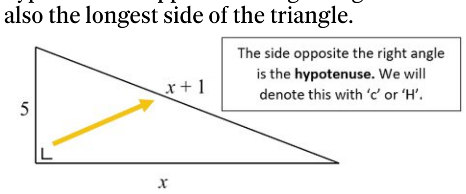
This section is a breakdown of a popular Fifth Year topic. Trigonometry is a common topic for Fifth Year as it typically appears in both Section A and Section B. The questions for Trigonometry are split into the two types; questions involving **right-angled triangles** and questions involving triangles that are **not right-angled**. For right-angled triangles, students need to be proficient at using Pythagoras' Theorem, sin, cos, tan, $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$. For non-right-angled triangles, students will need to know how to apply the formulas for the Sine Rule, the Cosine Rule and the Area of a Triangle. These formulas are in the Maths tables on page 16.

2016 Paper 2 – Question 6

The lengths of the sides of a right-angled triangle are 5, x and $x + 1$ as shown. Use the Theorem of Pythagoras to find the value of x.



Solution: In the above triangle, we will use c to denote the hypotenuse and then use b and c to denote the remaining sides. The hypotenuse is opposite the right-angle. It is also the longest side of the triangle.



$c^2 = a^2 + b^2$

$\Rightarrow (x + 1)^2 = (x)^2 + (5)^2$

$\Rightarrow (x+1)(x+1) = x^2 + 25$

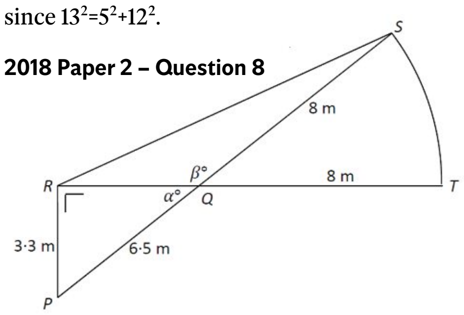
$\Rightarrow x^2 + x + x + 1 = x^2 + 25$

$\Rightarrow 2x = 24$

$\Rightarrow x = 12$

This should make sense. Don't forget that you can **verify** your answer. Substitute this answer into Pythagoras' Theorem. If correct, the theorem should balance. Here it does since $13^2=5^2+12^2$.

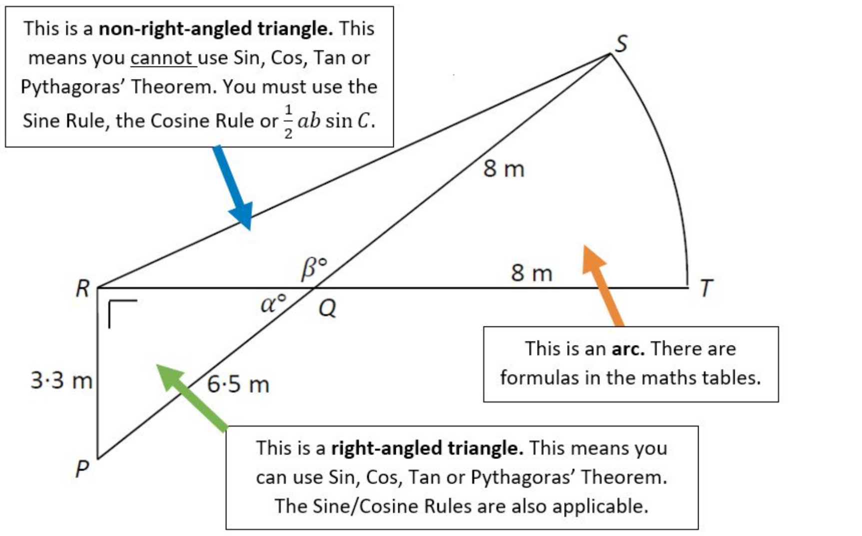
2018 Paper 2 – Question 8



I would recommend students to pause and identify each shape that they see in the above diagram. This will help them identify the correct formulae.

(a) Use the theorem of Pythagoras to find $|RQ|$.

We can use Pythagoras' Theorem since we



have two sides of a right-angled triangle.

$|PQ|^2 = |PR|^2 + |RQ|^2$

$|6.5|^2 = |3.3|^2 + |RQ|^2$

$|6.5|^2 - |3.3|^2 = |RQ|^2$

$|RQ|^2 = 31.36$

$|RQ| = \sqrt{31.36} = 5.6$ m

(b) Show that $a = 31°$, correct to the nearest degree.

Use $\text{Sin}\, a = \frac{Opposite}{Adjacent}$ to find the angle a.

You need to use $\sin^{-1}$ in order to find the angle.

$$\sin\alpha = \frac{3.3}{6.5}$$

$$\alpha = \sin^{-1}\left(\frac{3.3}{6.5}\right) = 30.51 = 31°.$$

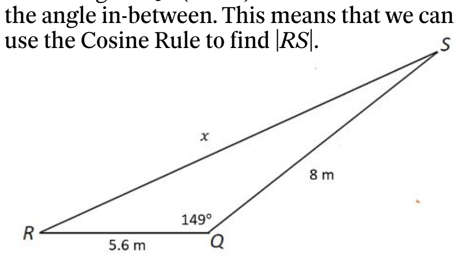
(c) Use the value of a given in part (b) to find the value of β.

There are 180° in a straight line.

$\beta = 180° - 31° = 149°$

(d) Use the Cosine Rule to find the length of [RS]. Give your answer correct to the nearest metre.

The triangle RQS (*below*) has two sides and the angle in-between. This means that we can use the Cosine Rule to find $|RS|$.



$a^2 = b^2 + c^2 - 2bc\cos A$

$|RS|^2 = 8^2 + 5.6^2 - 2(8)(5.6)\cos 149$

$|RS|^2 = 8^2 + 5.6^2 - 2(8)(5.6)\cos 149$

$|RS|^2 = 64 + 31.36 + 76.80219014$

$|RS|^2 = 172.16219014$

$|RS|^2 = \sqrt{172.16219014} = 13.121$

$|RS| = 13$ metres

Part (e) and (f) are both **Arc** questions. The formulas for both parts can be found in the Maths tables.

(e) SQT is a sector of a circle whose centre is Q. Find the length of the arc TS. Give your answer in metres, correct to one decimal place.

$$\text{Arc TS} = 2\pi r \times \frac{\theta}{360}$$

$$= 2\pi(8) \times \frac{31}{360}$$

$$= \frac{62\pi}{45} = 4.3284 = 4.3 \text{ metres.}$$

(f) Find the area of the sector SQT. Give your answer in square metres, correct to one decimal place.

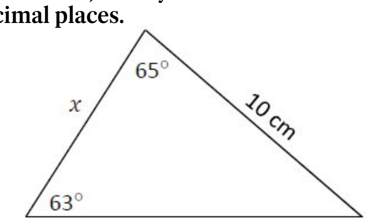
$$\text{Area} = \pi r^2 \times \frac{\theta}{360}$$

$$= \pi(8)^2 \times \frac{31}{360}$$

$$= \frac{248\pi}{45} = 17.3 \text{ metres squared.}$$

2017 Paper 2 – Question 6

(a) Find the distance x in the diagram below (not to scale). Give your answer correct to 2 decimal places.



Solution: Before any trigonometry question, check the triangle to see if you can work out any of the missing angles. Since the angles in a triangle add up to 180°, the missing angle above is 180° – 65° – 63° = 52°. We need this information to use the Sine Rule.

There are two scenarios for the Sine Rule:

Need two sides and one opposite angle

Need two angles and one opposite side

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

Here, $a = x$, $A = 52$, $b = 10$ and $B = 63$.

$$\frac{x}{\sin 52} = \frac{10}{\sin 63}$$

Cross-multiplying gives:

$x \sin 63 = 10 \sin 52$

$$x = \frac{10\sin 52}{\sin 63} = \frac{10}{\sin 63} = 8.84 \text{ cm.}$$

What can parents do to help?

Encouragement and praise: Students need to believe that they can succeed in Maths, especially at Ordinary Level. The Leaving Cert is an arduous and stress-filled year; the encouragement you give your son/daughter will help alleviate some of the pressure of this year.

Ask questions: Remind your child about the importance of asking questions. Students should not suffer in silence if they don't fully understand a topic. It's easier to ask a question in class than come up against a topic you do not understand on the day of the exam. As the saying goes, "the squeaky wheel gets the oil". There are no silly questions in a classroom, especially when you are trying to understand something tricky like Maths.

Practice: Reiterate the importance of practice. Maths is a practical subject that needs to be understood, not learnt off by heart.

Study plan: Students should have a study plan in place from now until June. There should be goals set each week and praise should be given when goals have been reached. The Leaving Cert is an academic marathon; every student out there needs people in the stands cheering them on.

Don't use negative language to describe Maths because it perpetuates the image that it is an 'impossible subject'. Most of mathematics is like following the steps of a simple recipe. It just takes a little patience and attention to detail.

Sometimes students choose Ordinary Level for strategic reasons and decide to focus their time on other subjects. Remind your son/daughter **not to neglect** maths and that they have to pass the subject. A minimum pass grade in Maths is a requirement for a large amount of third-level courses.

Advice for students in Fifth Year: how to stay motivated

Being self-motivated is the key to success in the Leaving Cert, especially for a practical subject like Maths, which rewards students' critical reasoning and logical thought process. The method is far more important than the answer and the marking scheme reflects this.

Practice makes perfect! Fifth year is the perfect year to achieve this. This journey starts with a single step so why not make that move in Fifth Year!

Set small achievable goals throughout the year. Having a **study plan** will help lighten the load of the subject. Students should not feel overwhelmed or intimidated by the course. Instead, students should focus on their little victories that they achieve in class and use this to fuel their learning. Once students get a taste of achieving their aims on a regular basis, it will motivate them to strive for even more.

Building blocks of knowledge: I often describe Maths as a Lego house of ideas. It is important for students to have a strong foundation in Maths because the Leaving Cert will test all facets of their ability. Thankfully, there comes a point during the year when a lightbulb goes off for the students and things begin to click. This 'Eureka moment' is the biggest indication of success. After this, the student will get to a stage where they feel comfortable and competent with the concepts of Maths. Working towards building a strong, fundamental foundation helps achieve this.

Prepare for a variety of different questions: Section A is made up of questions where students must exhibit their skills of mathematics. These short questions are more formula-heavy, and are more student friendly. Section B will test students' ability to translate their knowledge to real-world situations. Section B will challenge all students so practice is heavily encouraged in Fifth Year.

List of mistakes: Throughout the year, students should make a list of mistakes that they have made in class and in their class assessments. Homework and assessments are important learning opportunities for students because they enable students to recognise their most common mistakes. Albert Einstein said the definition of insanity is doing something over and over again and expecting a different result. Learn from these mistakes because you do not want to repeat them in the actual exam. Making mistakes is inevitable, but learning from your mistakes will ensure your success in Maths.